Generation of musical patterns through operads
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Multi-patterns as operations

An n-symmetric operad, or an operad for short, is a triple \((\mathcal{O}, v_n, 1)\) such that \(\mathcal{O}\) is a set decomposing as a disjoint union \(\mathcal{O} = \bigsqcup_{n \geq 0} \mathcal{O}(n)\), where
\[
v_n : \mathcal{O}(n) \times \bigotimes_{i=1}^{n} \mathcal{O}(m) \to \mathcal{O}(n + m - 1), \quad 1 \leq i \leq n.
\]
\(v_n\) is a map
\[v : \mathcal{O}(n) \times (\bigotimes_{i=1}^{n} \mathcal{O}(m)) \to \mathcal{O}(n + m - 1), \quad 1 \leq i \leq n.
\]
and \(\bigotimes_{i=1}^{n} \mathcal{O}(m)\) is an element of \(\mathcal{O}(1)\), called unit.

Multi-patterns as operations

Multi-patterns are operations that allow for the composition of musical patterns. For any \(m \geq 1\), an \(m\)-multi-pattern is a sequence of \(m\) patterns having the same length. For any \(m\)-multi-pattern \(m\),
- the multiplicity of \(m\) is \(m\);
- the arity \(|m|\) of \(m\) is the minimal arity among its patterns;
- the length \(\ell(m)\) is the common length of its patterns.

The operad of multi-patterns

The operad box model is a model to represent musical phrases by \(m\)-multi-patterns. Together with a scale and a root note, an \(m\)-multi-pattern denotes a musical phrase:
- Each pattern \(m\) denotes a monophonic phrase;
- Each integer \(m\) denotes a scale degree lasting one unit of time;
- Each \(c\) in \(m\) extends the duration of a note for one unit of time.

Random generation

Given a bud generating system \((\mathcal{O}, \mathcal{R}, i, \mathbb{C})\), it is possible to generate at random an element of \(\mathcal{O}\) by means of the following algorithm.

Algorithm

For any \(\mathcal{O}\), a \(\mathcal{R}\) is a finite set of colors, \(\mathcal{C}\) is the set of all rules of \(\mathcal{R}\), and \(\mathcal{O}\) is a finite subset of \(\mathcal{R}\), called set of rules, than a color of \(\mathcal{C}\), called initial color.

Colors play the role of non-terminal symbols, and each rule \((\mathcal{R}, \mathcal{C})\) can be seen as a production rule allowing us to replace an input having \(\mathcal{C}\) by color \(\mathcal{C}\) and by its attached input colors.

For any color \(\mathcal{C}\) in \(\mathcal{C}\), we shall denote by \(\mathcal{R}_{\mathcal{C}}\) the set of all rules of \(\mathcal{R}\) having \(\mathcal{C}\) as output color.

Random generation

Given a bud generating system \((\mathcal{O}, \mathcal{R}, i, \mathbb{C})\), it is possible to generate at random an element of \(\mathcal{O}\) by means of the following algorithm.

Algorithm

1. Title: Partial random generation algorithm
2. Inputs:
   - A bud generating system \(\mathcal{O} \rightarrow (\mathcal{O}, \mathcal{C}, R, i)\);
   - An integer \(n \geq 0\);
3. Algorithm:
   - Output: an element of \(\mathcal{O}\).
   - Set \(x = \text{the element } (\mathcal{O}, i, R)\);
   - Repeat \(n\) times:
     1. Pick a position \(i \in |x|\) at random;
     2. If \(R(x, i) = \emptyset\) then
        - Pick a rule \(r \in R(x, i)\) at random;
        - Set \(x = x / r\);
   - Returns \(x\).

Complete example

[Complete example details and code here]