Let \((M, \star)\) be a unitary magma, that is a set \(M\) endowed with a binary operation \(\star\) satisfying \(a \star 1 = a = 1 \star a\) for all \(a \in M\).

An \(M\)-clique of size \(n\) is a complete graph on the set \([n + 1]\) of vertices such that each arc \((x, y)\) is decorated by an element \(p(x, y) \in M\).

An arc \((x, y)\) is solid if \(p(x, y) \neq 1_M\).

Example
Here is a prime \(\mathbb{Z}/6\mathbb{Z}\)-clique of size 5. Only the solid arcs of \(p\) are shown:

Let \(C\) be the functor from the category of unitary magmas to the category of graded vector spaces defined by 
\[
C_M = \bigoplus_{n=0}^{\infty} M(n),
\]
where \(C_M(n)\) is the space generated by \(\text{solid dissections of polygons, and Lucas configurations.}\)

Substructures and quotients

In order to construct substructures of \(C_M\) whose bases are indexed by particular \(M\)-cliques, we use the following ideas:

**Idea A.** construct quotients of \(C_M\) by:
1. considering a family \(X\) of \(M\)-cliques;
2. setting \(R_X\) as the linear span of all the \(M\)-cliques of \(X\),
3. where \(R_X\) is an operad ideal of \(C_M\), the quotient 
\[
\frac{C_M}{R_X}
\]
is an operad on the linear span of \(R_X\);

**Idea B.** if \(R_1, R_2,\ldots, R_k\) are operad ideals of an operad \(C, R_1 + R_2 + \ldots + R_k\) is an operad ideal of \(C\) and \(C/R_1 + \ldots + R_k\) is a quotient of \(C\).

One obtains (among others) the operads fitting in the following diagram (arrows \(\rightsquigarrow\) resp. \(\Rightarrow\) are injective resp. surjective morphisms of operads):

The degree \(\text{deg}(p)\) of an \(M\)-clique \(p\) is the greatest degree of the vertices of the graph formed by the solid arcs of \(p\).

Let \(\mathcal{NC}\) be the operad of noncrossing configurations.

**Proposition**
The set
\[
\mathcal{NC}(M) = \bigoplus_{n=0}^{\infty} NC(M(n)),
\]
of all \(M\)-triangles is a minimal generating set of \(\mathcal{NC}\).

Hence, unlike \(C_M\), \(\mathcal{NC}\) is a binary operad. Moreover, \(\mathcal{NC}\) admits two particular properties. Indeed, \(\mathcal{NC}\) is:
1. the smallest suboperad of \(C_M\) containing all the \(M\)-triangles;
2. the biggest binary suboperad of \(C_M\).

**Proposition**
When \(M\) is finite, the Hilbert series of \(\mathcal{NC}(M)\) satisfies
\[
t_0 + (x_0^2 + 2x_0 - 3) + (x_0^2 + 2x_0 + 1) M_3(x_0),
\]
where \(m = \psi_M\).

Moreover, when \(M\) is finite, for all \(n \geq 2,
\[
\dim(\mathcal{NC}(M)(n)) = \sum_{i=0}^{n-2} M_{n-i-2}(M_3(M)),
\]
where \(m = \psi_M\).

A dual \(M\)-clique is a \(M\)-clique such that all edges are decorated by pairs \(\{a, b\} \subseteq M\), and all solid diagonals by pairs \(\{a, b\} \subseteq M^2\) with \(a \neq b\).

**Example**
The \(2\mathbb{Z}\)-clique is a dual \(Z\)-clique.

**Proposition**
When \(M\) is finite, the bases of \(\mathcal{NC}(M)\) are indexed by dual \(M\)-cliques.