Estimating the number of active flows in a data stream over a sliding window

Éric Fusy and Frédéric Giroire

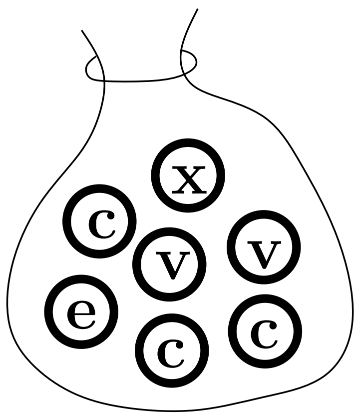
Algorithms Project, INRIA Rocquencourt

Overview

Estimation of large cardinalities

Cardinality of a multiset

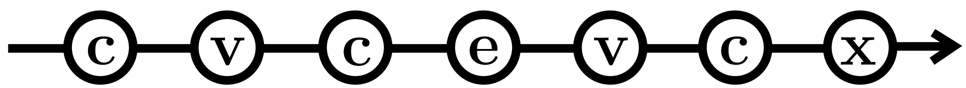
- Let $\mathcal{M}$ be a multiset,
  - N is the number of elements called the *size*
  - n the number of distinct elements called *cardinality*.

$\mathcal{M} =$ 

size $N = 7$

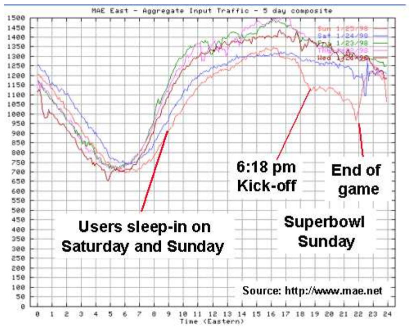
cardinality $n = 4$

elt.	e	v	c	x
mult.	**1**	**2**	**3**	**1**



- **Problem:** compute the cardinality n in one pass and with small auxiliary memory.

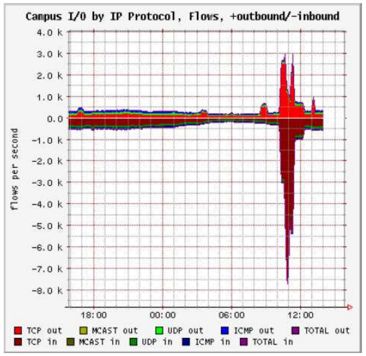
Surprisingly long list of applications



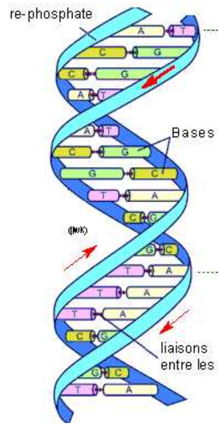
Traffic analysis

Linguistic

Very large multisets!



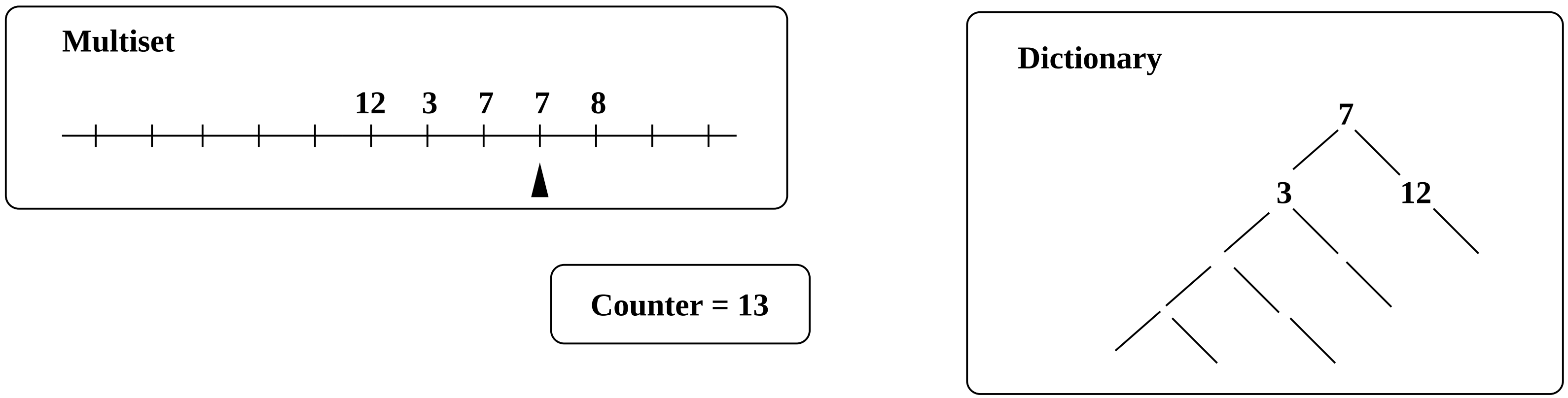
Detection of attacks



Analysis of the genome

Exact solution

- Maintain distincts elements already seen.



- One pass, but auxiliary memory of order n.
- **Information theory:** memory $\Omega(n)$ necessary

Probabilistic solution

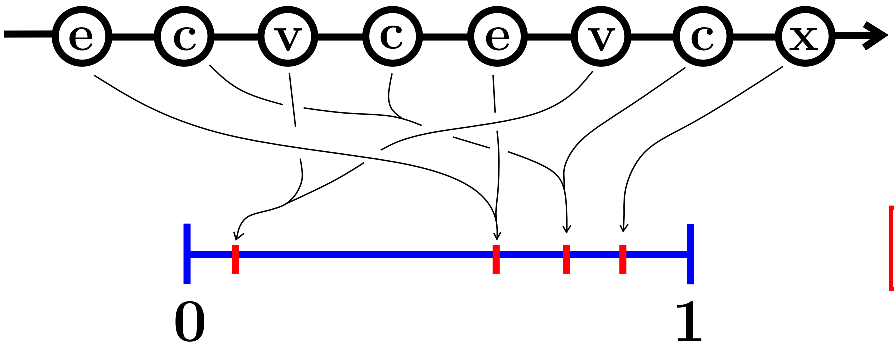
Crucial idea: relax the constraint of exact value of the cardinality. An estimate with good precision is sufficient for the applications.

Several algorithms have been proposed

- *Probabilistic Counting*, Flajolet and Martin 1983.
  LogLog Counting, Durand and Flajolet 2003.
- *Linear Counting*, Whang, Zanden and Taylor 1990.
- *Counting distinct elements in a data stream*, Bar-Yossef et al. 2002.

Probabilistic solution

- Elements of $\mathcal{M}$ are hached to $[0,1]$.



n **éléments distincts**

$$\Big\downarrow h$$

$$\boxed{n \textbf{ v.a. uniformes sur } [0,1]}$$

Probabilistic solution

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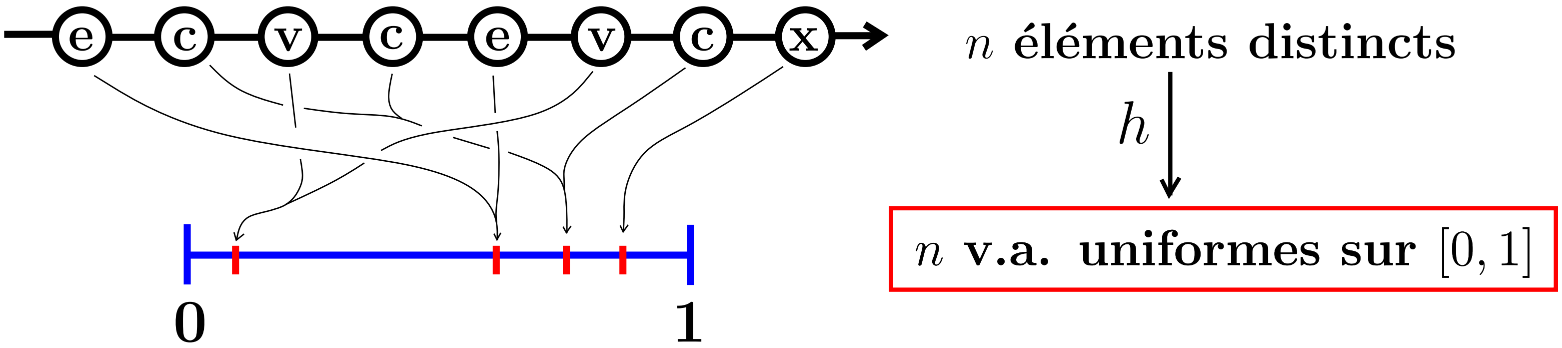

$$\mathbb{E}(\mathrm{Min}) = \frac{1}{n+1}$$

$\Rightarrow$ Idea: use the minimum to estimate the cardinality

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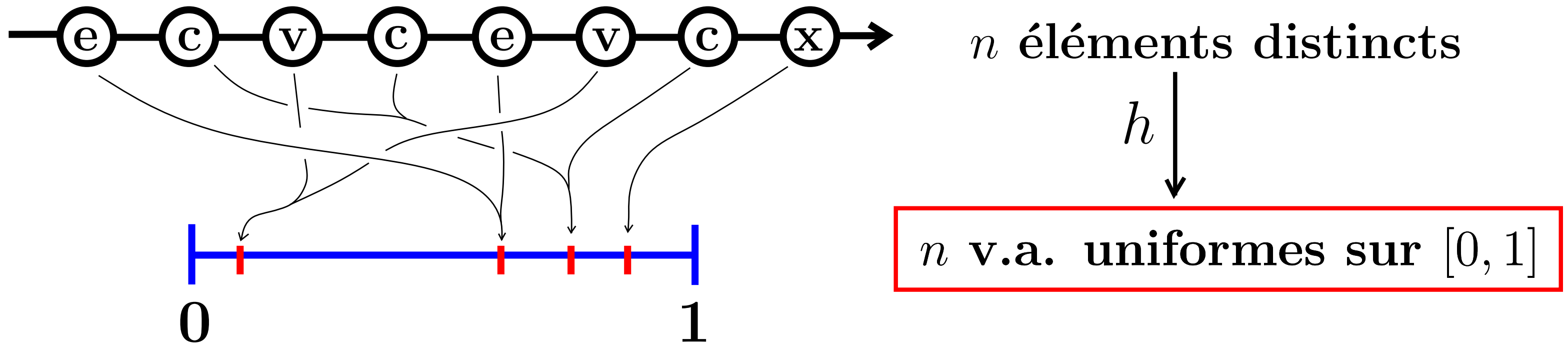

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$\Rightarrow$ Idea: use the minimum to estimate the cardinality

- The minimum is computed in one pass with constant memory

The algorithm MinCount

- Simulate m hashing functions
  $\Rightarrow m$ minima $M^{(1)}, \ldots, M^{(m)}$
- Estimate $= \alpha_m \times$ geometric mean of the $1/M^{(i)}$
- Relative error $\approx 1/\sqrt{m}$ for a memory of m words
  Accuracy of 4% with only 1kB of memory!
- If some buckets are empty (no minimum) use the number of empty buckets to estimate the cardinality

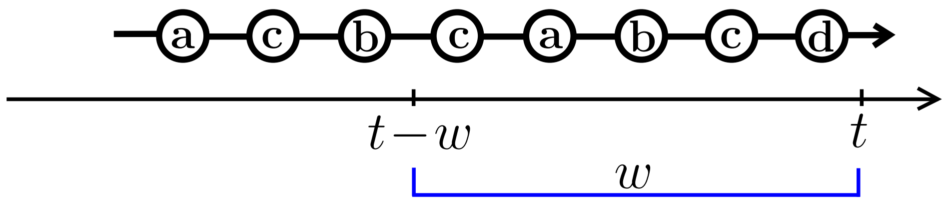
Counting over a sliding window

New context

- Telecom context: stream of IP packets passing by a router
- Each packet belongs to a flow (connection), identified by <source IP, destination IP>
- Elements of the multiset = packets
  Distinct elements of the multiset = flows
- Typical request: "What is the number of active flows over the last hour"

Sliding window

- Model studied by [Datar, Gionis, Indyk, Motwani]: "Maintaining Stream Statistics over a Sliding Window"
- Problem: At each time t, we want to estimate the number of flows over $[t - w, t]$.



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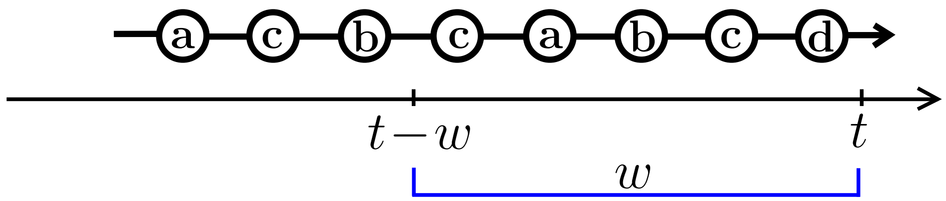

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  - New algorithm Sliding MinCount extends the (static) MinCount algorithm to the sliding window model
  - Complete analysis of the auxiliary memory
  - Validation on real traffic.

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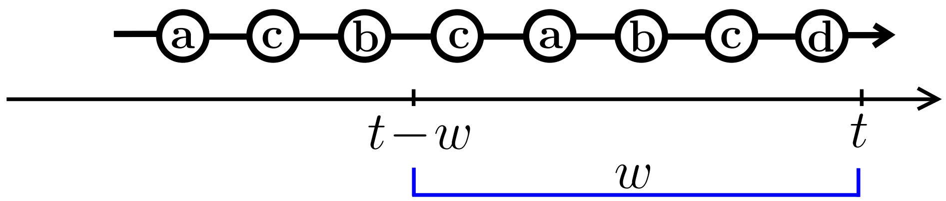

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The approach

- MinCount uses an estimate based on the minimum
- We have to maintain the minima of hashed values over a sliding window
- **Difficulty**: over a sliding window, outdated elements are discarded

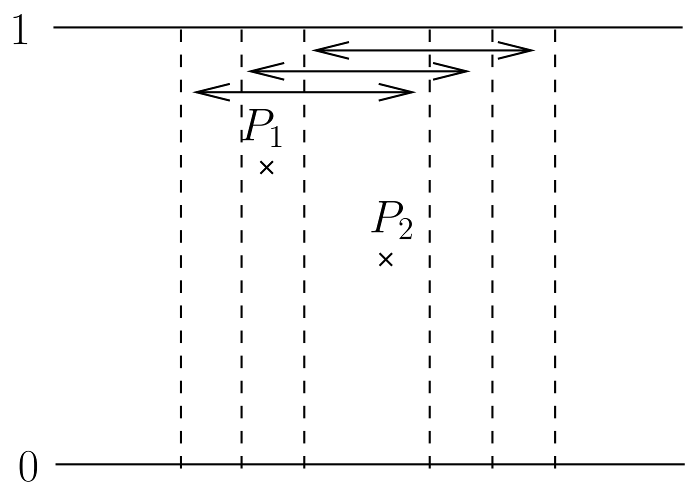
"How to remember if the discarded element has realized the minimum or not?!"

Maintain the minimum

- Solution: keep in memory the packets that may become a minimum in the future
- Crucial remark If $P_1 = (h_1, t_1)$ and $P_2 = (h_2, t_2)$ are two packets such that

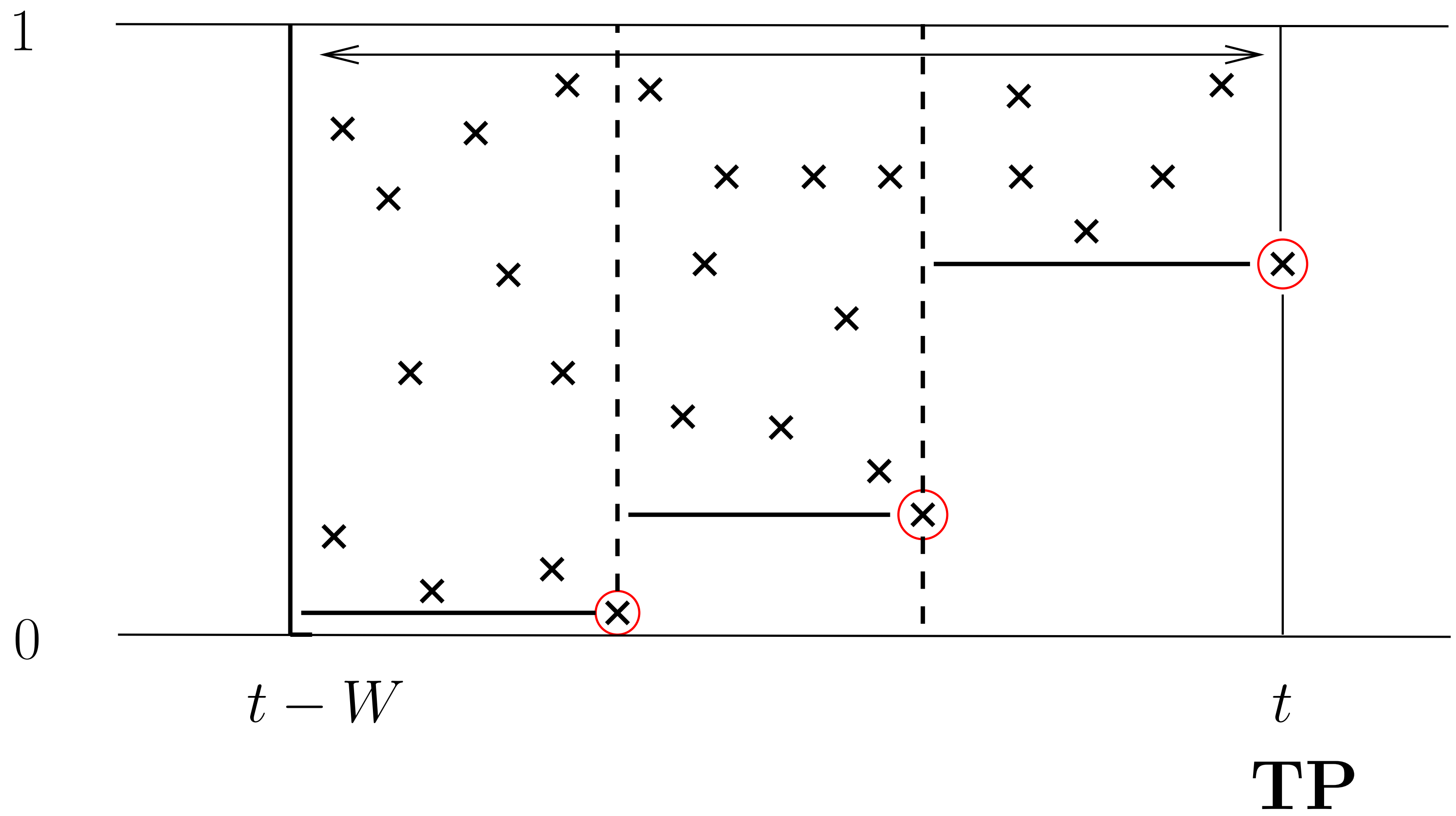
$$t_1 < t_2 \quad \text{and} \quad p_1 \geq p_2,$$

then P_1 can not become a minimum in the future.



The list of future possible minima

The future possible minima are the minimal records of hashed values taken in reverse chronological order.



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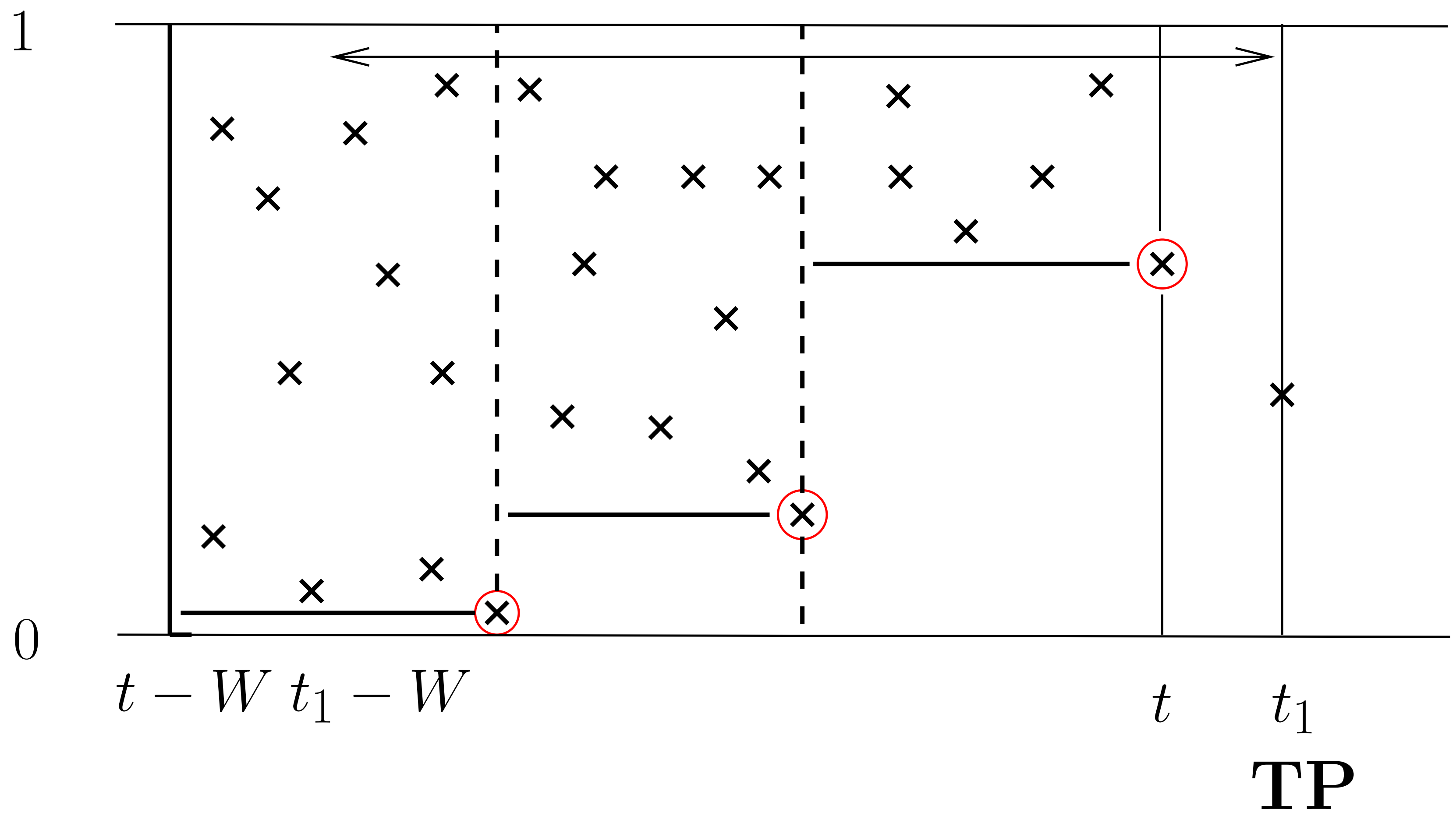

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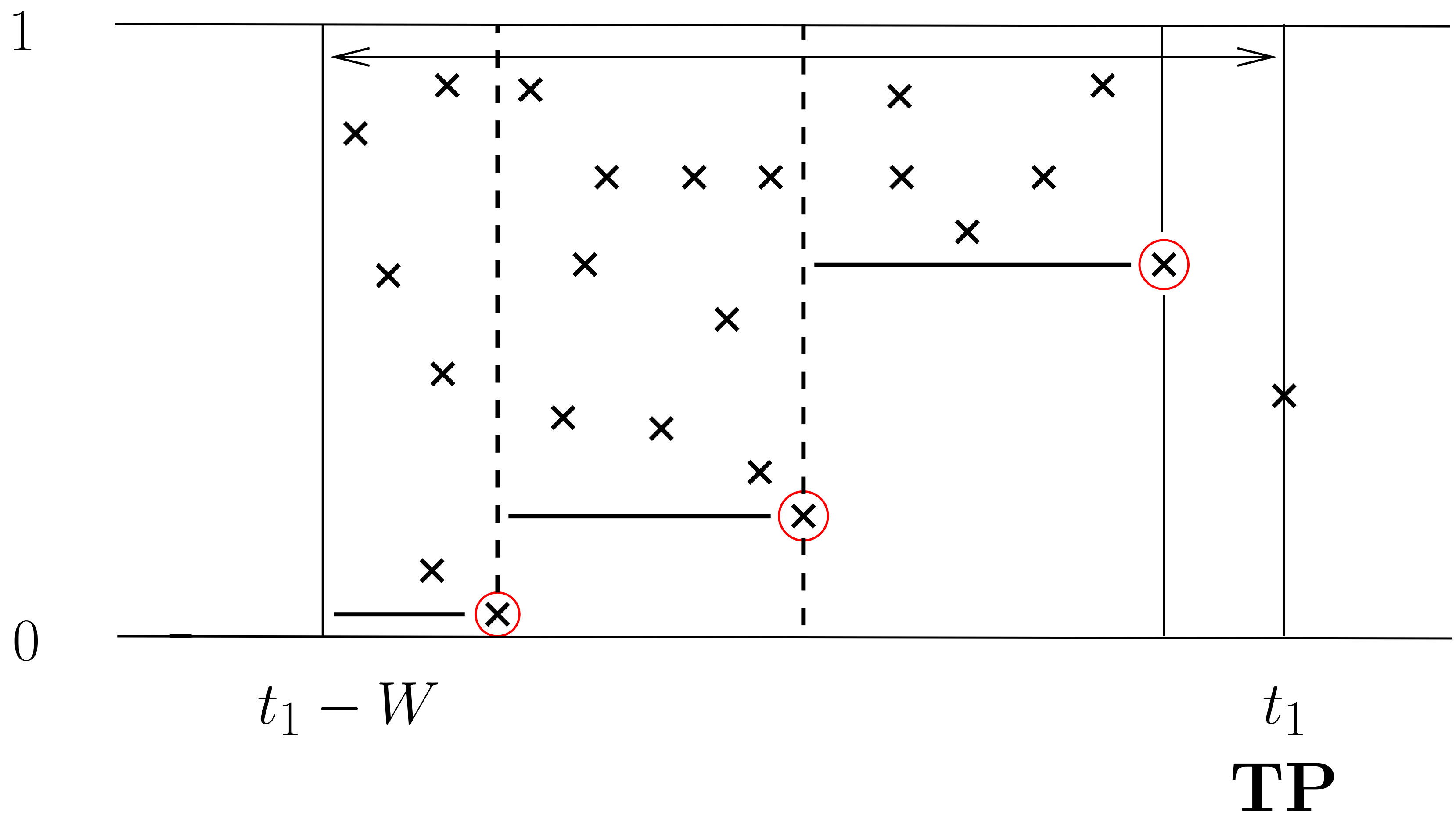

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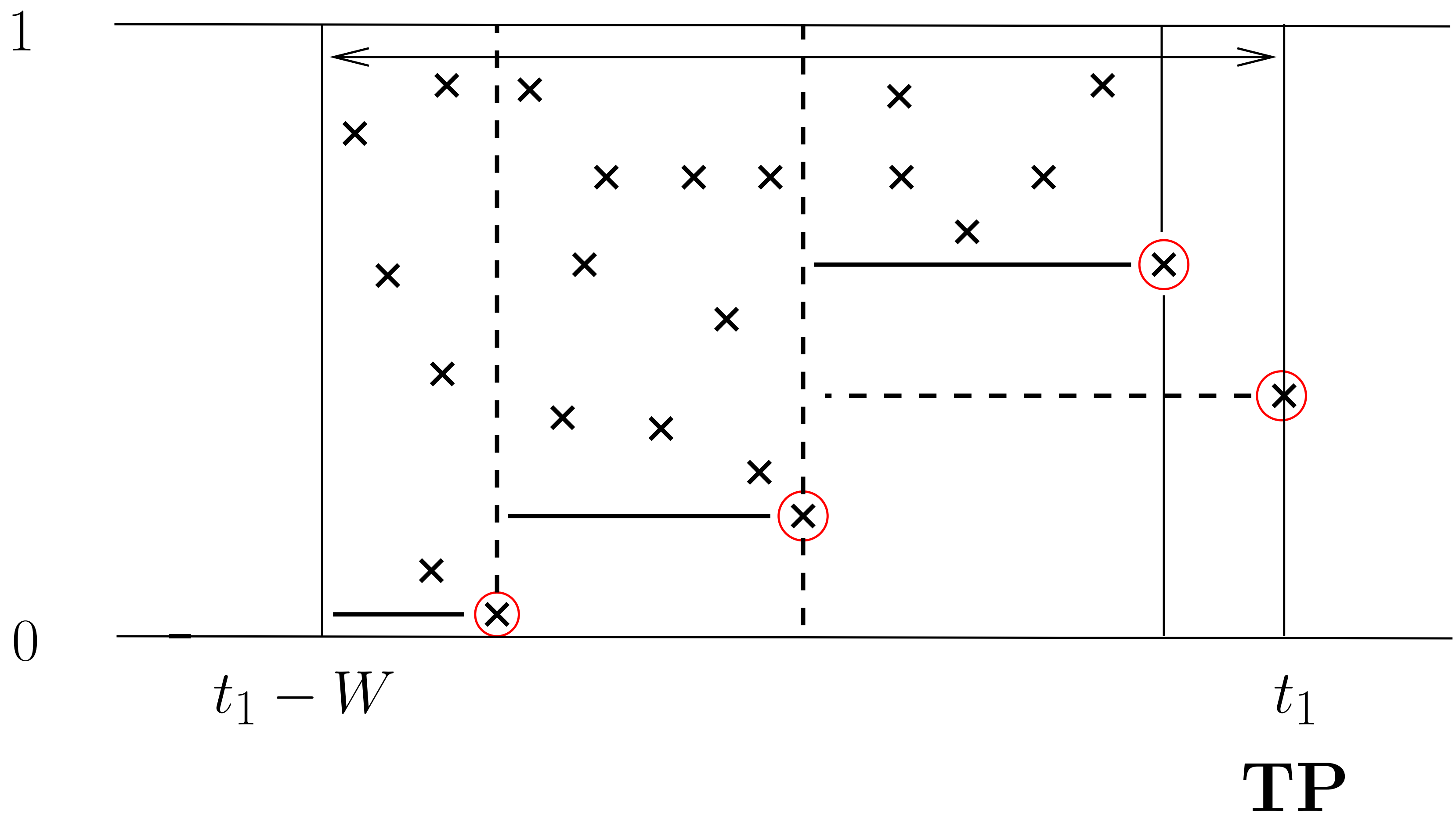

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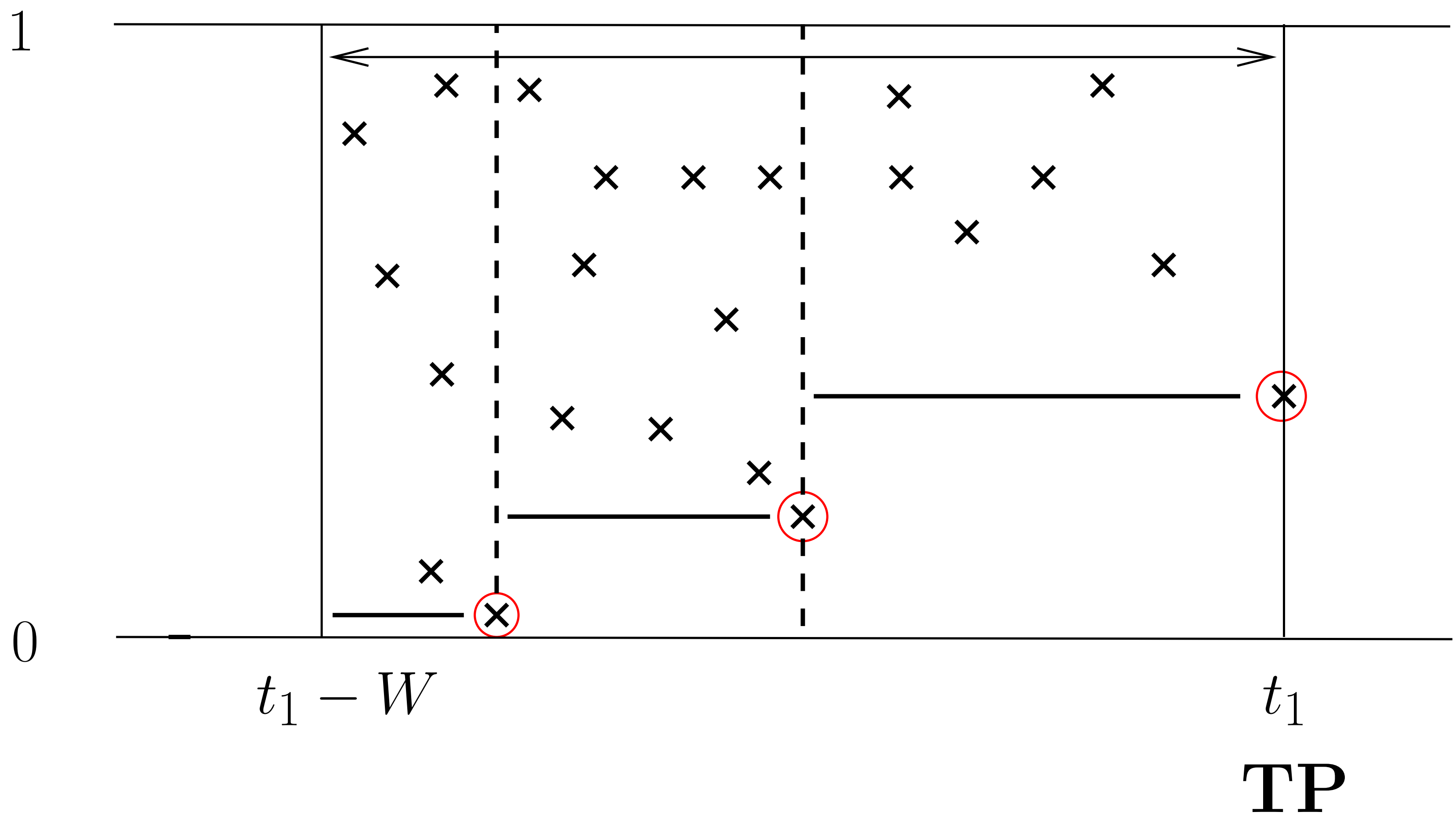
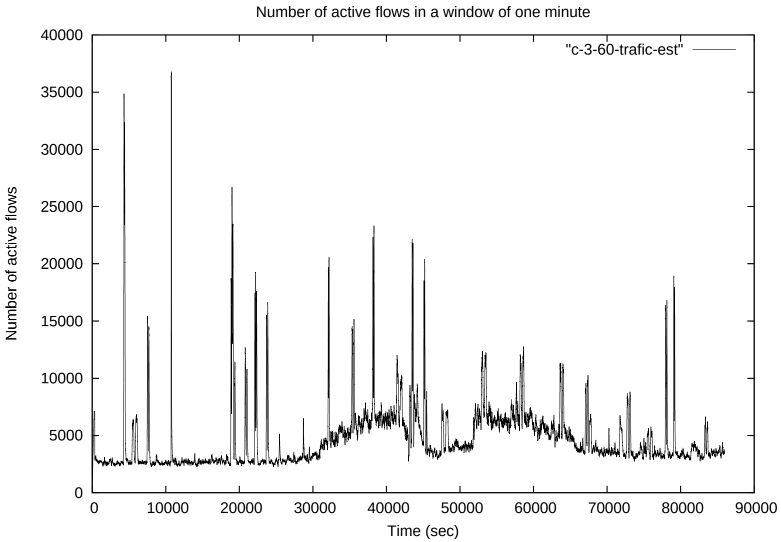
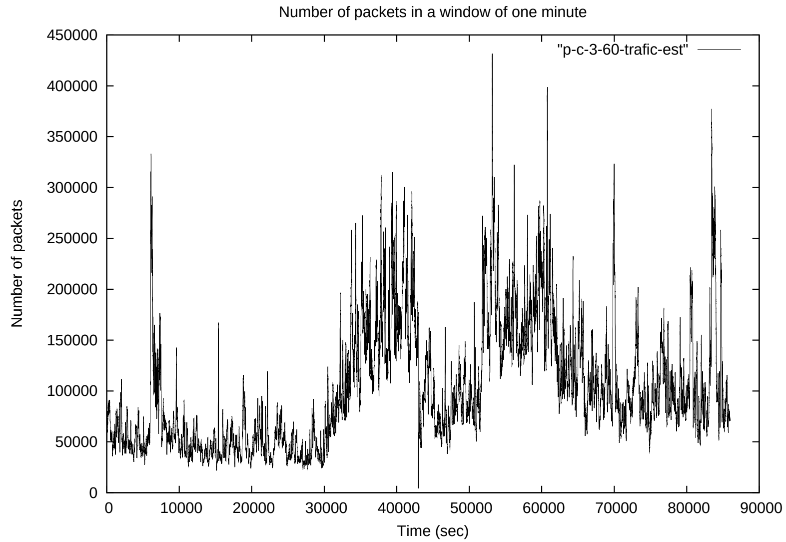


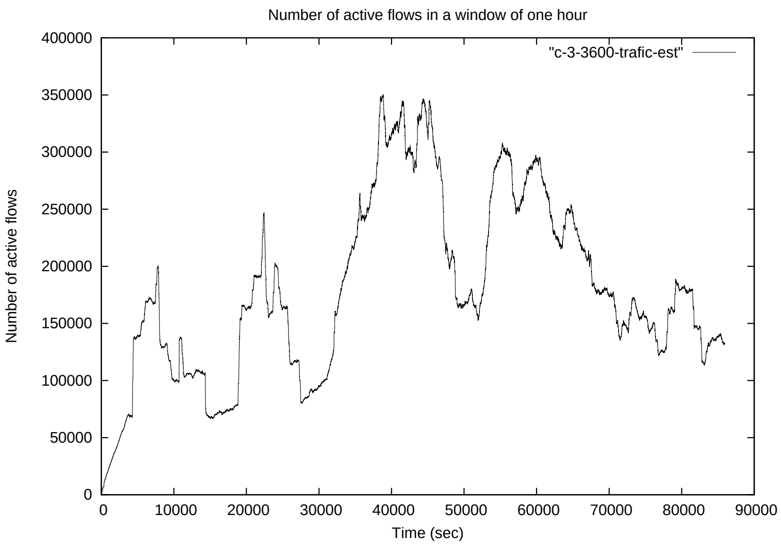
Results

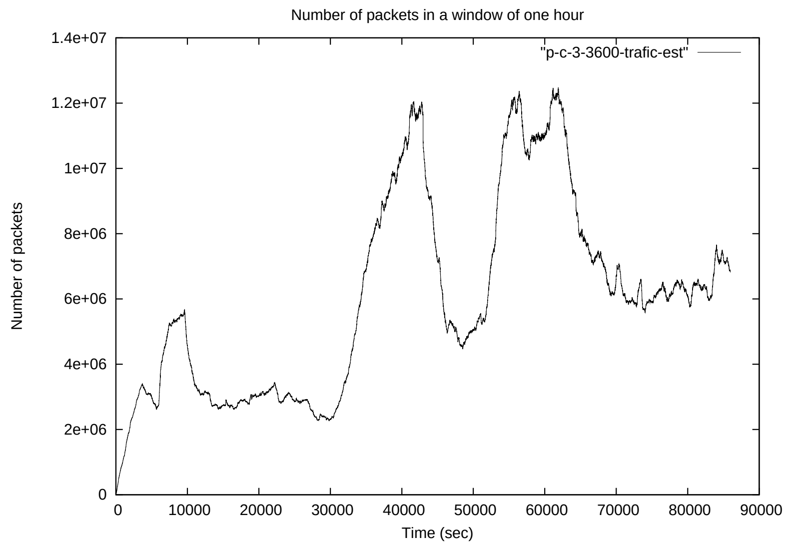
- Same accuracy as MinCount

Application to traffic monitoring

- 







Aggregated traffic of 400 machines at INRIA during one day. Comparison number of flows (Left) / number of packets (Right), for window of 1 hour (Top) / 1 minute (Bottom).