ACE

an Algebraic Combinatorics Environment

for the computer algebra system MAPLE

User's Reference Manual
Version 3.0

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Marne-la-Vallée
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ACE is a joint effort of a number of people from the University of Marne-la-Vallée. The project has been led since 1994 by Sébastien Veigneau (University of Marne-la-Vallée) who is grateful to all people having made up the version 3.0 of `ACE`. All the development has been done under supervision of Alain Lascoux (University of Marne-la-Vallée, CNRS) and Jean-Yves Thibon (University of Marne-la-Vallée). Many thanks also to Vincent Prosper for providing the necessary TEX and Perl knowledge to produce this documentation. The following persons have mainly contributed to the `ACE` project:

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Doing business without advertising is
like winking at a girl in the dark.
You know what you are doing,
but nobody else does.

New York Herald Tribune, October 30, 1956.

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```

Université de Marne-la-Vallée
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New features of release 3.0

Il ne faut pas dire que la science est utile
parce qu'elle nous permet de construire des machines.
Il faut dire : les machines sont utiles parce qu'elles
nous permettent de faire de la science.

H. Poincaré.

Users already familiar with `ACE` may be interested in the differences between versions 2.0 and 3.0. New users of `ACE` may skip this part and proceed with the next chapter. In the previous documentation, new features of release 2.0 with respect to version 1.0 of `ACE` were identified by a ✘ in the margin. This is not the case for this documentation because many changes have been ✘ made. New features of release 3.0 include:

- **New package** `COMP`: this package provides functions to work with compositions. Available functions deal with comparisons, major indexes, conjugates, lists of compositions.
- **New package** `FREE`: this package provides functions to work with the free algebra, that is the algebra of words. It is also possible to work in the plactic monoid, *i.e.* the free algebra of words modulo the plactic relations.
- **New package** `PART`: this package provides functions to work with partitions. Available functions deal with border, conjugate, diagonals, Frobenius notation, hooks, etc. of a partition, and also with lists of (skew) partitions.
- **New package** `SFA`: this is an extension of the `SYMF` package which gives a way to handle symmetric functions viewed as operators acting on the ring of polynomials with real coefficients. It corresponds to an implementation of the Λ-ring theory.
- **New possibilities** in the `SYMF` package: it is from now possible to add new bases of the ring of symmetric functions, by the mean of two new functions `SfAddBasis` and `SfDualBasis`.
- **Many function names** have been changed, many functions have disappeared, either becoming internal functions or being included as options of other more general ones. Function names have been strictly normalized.
- **New functions and algorithms** have been included in already existing packages in order to improve their efficiency. This is in particular the case for the `SG` (symmetric group), the `TAB` (tableaux) and `SYMF` (symmetric functions) packages.

- **Heavy modifications** in the coding of the elements of the different algebras implemented in `ACE`. Elements are now coded as sums of indexed names instead of sums of lists for some compatibility reasons with *Maple V.*4.
- **Heavy internal modifications** in the `CG` (classical groups) package have been performed in order to get it more efficient.
- **ACE installation** has been completely rebuilt in order to use new possibilities with respect to the development of packages, in particular in *Maple V*.4.
- **A test protocol** has been included in `ACE` in order to have the possibility to test whether the installation has been successful or not. It corresponds to the automatic production of test files (including more than 1000 tests) that you should read to test a specific package in `ACE`.
- **Removed package** `SHRI`: this package has been removed from the `ACE` system.
- **Removed package** `FOCK`: this package has been removed from the core of the `ACE` system. However, it is still available as a satellite package using `ACE` functionalities.
- **Some bugs** have been fixed.
- **A new documentation** is included in `ACE` 3.0, available both in postscript format and in the HTML hypertext language.
- **The full system** is available on CD-ROM, including related papers (articles, theses), a mathematical manual explaining many algorithms used in `ACE` and how to experiment with `ACE`, the HTML and postscript documentations. It also contains all sources, all installed (compiled) versions of `ACE` 3.0, together with the old versions of `ACE`.
- **Informations** related to the `ACE` CD-ROM can be obtained from:

  `http://phalanstere.univ-mlv.fr/~ace`

  or by email at:

  `ACE@univ-mlv.fr`.

  In any case, you can contact Alain Lascoux directly by email at:

  `Alain.Lascoux@univ-mlv.fr`.

Preface

ACE is a *Maple* library devoted to Algebraic Combinatorics computations. The project started on April 1994, at the University of Marne-la-Vallée, with the development of three *Maple* packages, dealing respectively with the symmetric group, Schubert polynomials and noncommutative symmetric functions. These packages were available in July 1994. We made up our minds to develop a more general project about algebraic combinatorics in October 1994. The aim was to build an "easy to use" Algebraic Combinatorics Environment. We have chosen *Maple* because this system is widely used by combinatorists and has a convenient user's interface. We had in mind to develop an efficient toolbox of functions for research in algebraic combinatorics. The interface had to be very flexible for first experimentations, whereas it may be more interesting to use some specific programs for large scale computations: for instance, we refer the reader to the `SYMMETRICA` system that is well suited for such computations [3].

We also refer the reader to the new system μ-EC (`http://phalanstere.univ-mlv.fr/~muec`) developped by Vincent Prosper (University of Marne-la-Vallée) which includes the `SYMMETRICA` system (to compute with symmetric functions) and can be seen as an environment of computation in combinatorics for the computer algebra system `MuPAD`. The μ-EC system includes many functions available in `ACE`.

The story started in 1994. `ACE` 1.0 was divided into various packages dealing with specific objects such as noncommutative symmetric functions, Schubert polynomials, tableaux or Hecke algebra, each package having the ability to use some functions defined in the other packages. The first version of `ACE` was available for any MS-DOS or UNIX system, for *Maple V* Release 1, 2 and 3.

The next version 2.0 was available since June 1996 for any MS-DOS, UNIX and MACINTOSH system. It provided new packages and new functions in the already existing packages. Several people had been involved in the project in order to develop new programs as for instance to work with the hyperoctahedral group and with several algebras associated to the hyperoctahedral groups B_n, together with their realizations as algebras of operators on the ring of polynomials. Moreover, a package has been developed to work with the representation theory of classical Lie groups (symplectic and orthogonal groups), whereas a new package was devoted to computations in the polynomial ring $\mathbb{Z}[x_1, \ldots, x_n]$ regarded as a free module of rank $n!$ over the ring $\mathbf{Sym}[x_1, \ldots, x_n]$ of symmetric polynomials, and another one was devoted to calculations in the Fock space representations of the quantum affine algebras $U_q(\widehat{sl}_n)$.

The third version of `ACE`, that is `ACE` 3.0, has been released at the beginning of 1998. Many changes have been made in order to get a coherent system devoted to computations in algebraic combinatorics. Many function names have been changed, many functions have disappeared, either becoming internal functions or being included as options of other more general ones. Function names have been strictly normalized.

`ACE` 3.0 has thus less functions but more functionalities. In particular, several packages have been included such as two packages to handle partitions and compositions of integers, one package to work in the free algebra, one package dealing with symmetric functions on different alphabets.

The ACE system, version 3.0, is now available for *Maple V* Release 1, 2, 3 and 4, for any MS-DOS, UNIX or MACINTOSH system.

This documentation describes how this new version of ACE can be installed and how the new user can make his first steps towards an efficient use of the environment. It gives a complete documentation about all mathematical available functions.

I hope this system will be useful for further developments in computer science in relation with algebraic combinatorics. I would like to thank you for your interest in ACE, "*quant à moi, je vais désormais voler vers d'autres horizons...*".

Sébastien Veigneau,
Université de Marne-la-Vallée, June 2, 1998.

```
############################################################
# Algebraic Combinatorics Environment,  3.0,  June 2, 1998 #
############################################################
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# Email: ACE@univ-mlv.fr                                   #
# URL: http://phalanstere.univ-mlv.fr/~ace                 #
#####################################################
# ACE  Algebraic Combinatorics Environment         #
#                                                  #
# SFA  Symmetric Functions on different Alphabets  #
# FM   Free Module over symmetric polynomials      #
# NCSF NonCommutative Symmetric  Functions         #
# CLG  Characters of Linear Groups                 #
# SP   Schubert Polynomials                        #
# SYMF SYMmetric Functions                         #  __/__/ __/_/_/ __/_/__/
# TAB  TABleaux                                    #  __/ __/ __/     __/
# FREE FREE algebra                                #  __/_/__/ __/     __/_/
# BNA  hyperoctahedral Algebras                    #  __/   __/ __/     __/
# SGA  Symmetric Group Algebra                     #  __/    __/ __/_/_/ __/_/__/
# NCA  NilCoxeter Algebra                          #
# IDCA IDCoxeter Algebra                           #               _/_/_/    _/_/_/
# HEKA HEcKe Algebra                               #                _/_/    _/  _/
#                                                  #           _/_/_/ _/ _/_/_/
# BN   hyperoctahedral groups                      #
# SG   Symmetric Groups                            #
# CG   Classical Groups                            #
#                                                  #
# PART PARTitions                                  #
# COMP COMPositions                                #
#                                                  #
# ERR  ERRors                                      #
# TEX  TEX output                                  #
# TYP  TYPe checking                               #
#######################
```

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- BnGaOnPol - action of an element of the B_n-algebra . . . 259
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- SfPlethysm - plethysm of symmetric functions . . . 290
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