Random generation of musical patterns through operads

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Meetup - Creative Code Paris

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Outline

Musical patterns

Scales

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- Examples -

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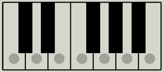

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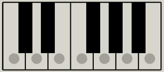
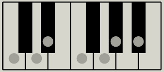


- The C minor scale is the subset $\{\mathbf{0}, 2, 3, 5, 7, 8, 10\}$.



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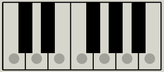

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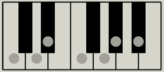

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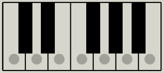

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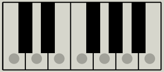

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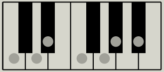

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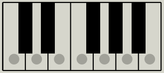

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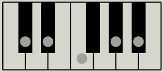

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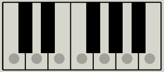

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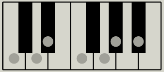

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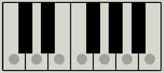

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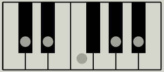

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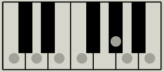

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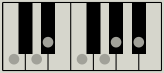
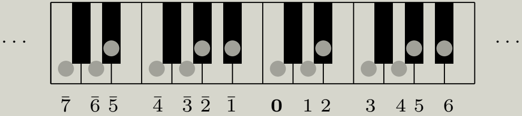


Here is the correspondence between degrees and notes:



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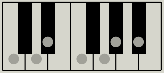

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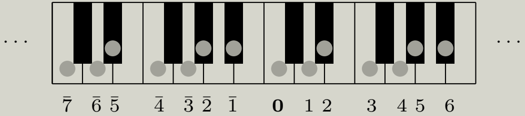

Let $\ell(\boldsymbol{\lambda})$ be the number of tones of $\boldsymbol{\lambda}$.

- Degree 0 encodes the root note of $\boldsymbol{\lambda}$.
- Degree $d + \ell(\boldsymbol{\lambda})$ encodes a note an octave above the one of d.
- Degree $d - \ell(\boldsymbol{\lambda})$ encodes a note an octave below the one of d.

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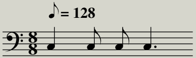
interpreted with a tempo of 128 gives



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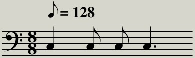

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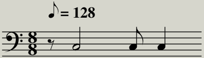

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The previous pattern, interpreted in the A minor scale with a tempo of 192 gives

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$$\begin{bmatrix} 0 & \square & \square & \square \\ 2 & \square & \square & \square \\ 4 & \square & \square & \square \end{bmatrix}$$
is a 3-multi-pattern, encoding a triad chord in a scale of length 7.

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The 2-multi-pattern

$$\mathbf{p} := \begin{bmatrix} 0 & \square & \square & \square & 4 & \square & 3 & \square & 4 \\ 4 & \square & \square & 0 & \square & \square & 3 & \square & 10 \end{bmatrix}$$

interpreted with a tempo of 128 gives

Operations on patterns

Composition of degree patterns

Let us consider the degree patterns

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This is the interpretation of the degree pattern

$$\mathbf{d}'' = 0\,2\,4 \;\; 1\,3\,5 \;\; 2\,4\,6 \;\; 3\,5\,7 \;\; 4\,6\,8 \;\; 5\,7\,9 \;\; 6\,8\,10 \;\; 7\,9\,11.$$

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The composition of two degree patterns $\mathbf{d}$ and $\mathbf{d}'$ is the degree pattern

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All the following degree patterns are interpreted in the A harmonic minor scale and with a tempo of 128.

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$$\mathbf{d} \odot \mathbf{d}' := \left(\mathbf{d}_1 + \mathbf{d}'_1, \ldots, \mathbf{d}_1 + \mathbf{d}'_m, \quad \mathbf{d}_2 + \mathbf{d}'_1, \ldots, \mathbf{d}_2 + \mathbf{d}'_m, \quad \ldots, \quad \mathbf{d}_n + \mathbf{d}'_1, \ldots, \mathbf{d}_n + \mathbf{d}'_m\right),$$

where $n := |\mathbf{d}|$ and $m := |\mathbf{d}'|$.

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All the following degree patterns are interpreted in the A harmonic minor scale and with a tempo of 128.

- $0213 \odot 0\bar{1}1 = 0\bar{1}1\ 213\ 102\ 324$

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Composition of rhythm patterns

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$$\mathbf{r} := \blacksquare\blacksquare\square\blacksquare\square\square \quad \text{and} \quad \mathbf{r}' := \square\blacksquare\blacksquare.$$

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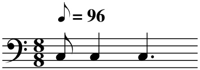
The interpretation of $\mathbf{r}$ and $\mathbf{r}'$ with a tempo of 96 are respectively



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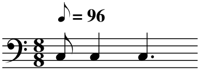

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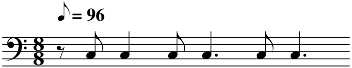

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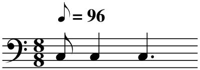

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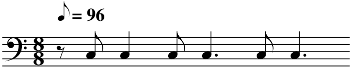

This is the interpretation of the rhythm pattern

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Composition of patterns and multi-patterns

These two previous operations extend to patterns.

The composition of two patterns $(\mathbf{d}, \mathbf{r})$ and $(\mathbf{d}', \mathbf{r}')$ is the pattern

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By using the concise notation for patterns,

$$3\square\square 1 2 0 1 \square \bar{1} \quad \odot \quad 1 \square 0 \quad = \quad 4 \square 3 \;\; \square\square \;\; 2 \square 1 \;\; 3 \square 2 \;\; 1 \square 0 \;\; 2 \square 1 \;\; \square \;\; 0 \square \bar{1}.$$

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- Example -

By using the matrix notation for 2-multi-patterns,

$$\begin{bmatrix} \square & 0 & 5 \\ \bar{3} & \square & 1 \end{bmatrix} \odot \begin{bmatrix} 2 & \square & \square & \square \\ \square & \square & \bar{1} & \square \end{bmatrix} = \begin{bmatrix} \square & 2 & \square & \square & \square & 7 & \square & \square & \square \\ \square & \square & \bar{4} & \square & \square & \square & \square & 0 & \square \end{bmatrix}.$$

Partial compositions

There is a refined a variant of the composition obtained my composing the second multi-pattern only onto one beat of the first one.

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Here, m-multi-patterns are therefore operators that can be composed through the $\circ_i$.

Random generation

A first simple algorithm

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- Algorithm RHM (Random Hook Monochrome generation) -

- Input:
  1. A finite set $\mathcal{R}$ of m-multi-patterns;
  2. An integer $\alpha \geqslant 0$.
- Output: an m-multi-pattern.

1. Set $\mathbf{m}$ as the m-multi-pattern $(0, \ldots, 0)$;
2. Repeat α times:
   2.1 Pick a position $1 \leqslant i \leqslant |\mathbf{m}|$ at random;
   2.2 Pick an m-multi-pattern $\mathbf{m}'$ of $\mathcal{R}$ at random;
   2.3 Set $\mathbf{m} := \mathbf{m} \circ_i \mathbf{m}'$;
3. Returns $\mathbf{m}$.

A first simple algorithm

- Example -

Consider the input data

$$\mathcal{R} := \left\{ \begin{bmatrix} \square & 1 & 0 & \square & 1 & 0 \\ 0 & 0 & 0 & 0 & \square & \square \end{bmatrix}, \begin{bmatrix} 1 & \bar{1} \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & \square \\ 0 & \square \end{bmatrix} \right\}$$

and $\alpha := 4$.

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The interpretation of this multi-pattern in the A minor pentatonic scale with 128 as tempo is

A possible problem

Assume that $\mathcal{R}$ contains a multi-pattern

$$\begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix}.$$

Then, at each step of Algorithm RHM, each partial composition increases by 4 some degree of the current multi-pattern.

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A problem can occur since the degrees of the generated patterns can be too high (or, analogously, too low), and cannot be interpreted as listenable notes.

Colored multi-patterns

A solution to this problem consists in protecting some positions of the pattern against some partial compositions.

For this, we consider a finite set $\mathfrak{C} := \{\mathtt{c}_1, \dots, \mathtt{c}_r\}$ whose elements are called colors, and augment multi-patterns with such colors.

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A colored multi-pattern is a triple $(a, \mathbf{m}, u)$ where

- $a \in \mathfrak{C}$ is the output color;
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- Example -

The triple

$$\left(\mathtt{c}_2, \begin{bmatrix} 0 & \square & 2 & \square & \square & 0 \\ 2 & \square & \square & 4 & \square & 0 \\ 4 & \square & \square & \square & 6 & 0 \end{bmatrix}, \mathtt{c}_3\mathtt{c}_1\mathtt{c}_1\right)$$

is a colored 3-multi-patterns.

Colored partial composition

The partial composition of two colored m-multi-patterns $(a, \mathbf{m}, u)$ and $(a', \mathbf{m}', u')$ at position $1 \leqslant i \leqslant |\mathbf{m}|$ is defined only if $a' = u_i$ and is the colored m-multi-pattern

$$(a, \mathbf{m}, u) \circ_i (a', \mathbf{m}', u') := (a, \mathbf{m} \circ_i \mathbf{m}', u \circ_i u'),$$

where $u \circ_i u'$ is the word obtained by replacing the i-th letter of u by u'.

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- Examples -

Let

$$x := \left(\mathtt{c}_1, \begin{bmatrix} \square & 1 & 1 & \square \\ 0 & \bar{2} & \square & \square \end{bmatrix}, \mathtt{c}_1\mathtt{c}_2\right) \quad \text{and} \quad x' := \left(\mathtt{c}_2, \begin{bmatrix} 3 & 0 & 0 \\ \bar{1} & \bar{1} & 1 \end{bmatrix}, \mathtt{c}_2\mathtt{c}_3\mathtt{c}_2\right)$$

be two colored 2-multi-patterns.

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Among others, the partial composition $x \circ_2 x'$ is well-defined, and $x \circ_1 x'$ is not authorized.

- Theorem -

For any $m \geqslant 1$, the set of all colored m-multi-patterns endowed with the partial composition maps $\circ_i$ is a colored operad.

Random generation algorithm

The size $|x|$ of a colored multi-pattern $x := (a, \mathbf{m}, u)$ is the size of $\mathbf{m}$.

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Given a finite set $\mathcal{R}$ of colored m-multi-patterns and a color $a \in \mathfrak{C}$, let $\mathcal{R}_a$ be the subset of $\mathcal{R}$ of the colored multi-patterns having a as output color.

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- Algorithm RH (Random Hook generation) -

- Input:
  1. A finite set $\mathcal{R}$ of colored m-multi-patterns;
  2. A color $a \in \mathfrak{C}$;
  3. An integer $\alpha \geqslant 0$.
- Output: an m-multi-pattern.

1. Set x as the colored m-multi-pattern $(a, (0, \ldots, 0), a)$;
2. Repeat α times:
   - 2.1 Pick a position $1 \leqslant i \leqslant |x|$ at random;
   - 2.2 Set b as the i-th input color of x;
   - 2.3 If $\mathcal{R}_b \neq \emptyset$:
     - 2.3.1 Pick a color m-multi-pattern x' of $\mathcal{R}_a$ at random;
     - 2.3.2 Set $x := x \circ_i x'$;
3. Returns the m-multi-pattern of x.

Random generation algorithm

- Example -

Consider the input data

$$\mathcal{R} := \left\{ \left(\mathtt{c}_1, \begin{bmatrix} \square & 1 & 0 & \square & 1 & 0 \\ 0 & 0 & 0 & 0 & \square & \square \end{bmatrix}, \mathtt{c}_1\mathtt{c}_1\mathtt{c}_2\mathtt{c}_1 \right), \left(\mathtt{c}_1, \begin{bmatrix} 1 & \bar{1} \\ 0 & 2 \end{bmatrix}, \mathtt{c}_2\mathtt{c}_1 \right), \left(\mathtt{c}_2, \begin{bmatrix} 0 & \square \\ 0 & \square \end{bmatrix} \mathtt{c}_3 \right) \right\},$$

$a = \mathtt{c}_1$, **and** $\alpha := 4$.

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Here is a possible execution trace of Algorithm RH:

$$\left(\mathtt{c}_1, \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \mathtt{c}_1 \right)$$

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Examples

Some tool patterns

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All these patterns can be altered by putting some colors $\mathtt{c}_2$ as input colors to prevent further compositions. The color $\mathtt{c}_2$ is a sink color: no colored multi-pattern has $\mathtt{c}_2$ as output color.

We work with the A minor harmonic scale with a tempo of 192.

Let

$$\mathcal{R} := \left\{ \left(\mathtt{c}_1, \begin{bmatrix} \bar{1} & \square & 0 & \square & 1 & \square \\ \square & 1 & \square & 0 & \square & \bar{1} \end{bmatrix}, \mathtt{c}_1\mathtt{c}_1\mathtt{c}_1 \right) \right\}.$$

Algorithm RH produces, for $\alpha = 20$, the phrase

We work with the A pentatonic scale with a tempo of 192.

Let

$$\mathcal{R} := \left\{ \left(\mathtt{c}_1, \begin{bmatrix} 3 & 4 & \square & 3 & 2 & 0 & 1 & 2 & \square & 0 & \square & \square \\ \bar{5} & \bar{5} & \bar{5} & 0 & \square & \square & \square & \square & 0 & 0 & 0 & 0 \end{bmatrix}, \mathtt{c}_2\mathtt{c}_2\mathtt{c}_2\mathtt{c}_1\mathtt{c}_1\mathtt{c}_1\mathtt{c}_1\mathtt{c}_1 \right) \right\}.$$

Algorithm RH produces, for $\alpha = 8$, the phrase

We work with the A harmonic minor scale with a tempo of 192.

Let

$$\mathcal{R} := \left\{ \left(\mathtt{c}_1, \begin{bmatrix} 0 & \square & 2 & \square & 4 & \square & 7 & \square & \square & 4 & 2 \\ 0 & \square & 2 & \square & \bar{3} & \square & \bar{7} & \square & \square & 0 & 0 \end{bmatrix}, \mathtt{c}_1\mathtt{c}_1\mathtt{c}_2\mathtt{c}_2\mathtt{c}_1\mathtt{c}_1 \right) \right\}.$$

Algorithm RH produces, for $\alpha = 16$, the phrase

We work with the A Hirajoshi scale $\{0, 4, 5, \mathbf{9}, 11\}$ with a tempo of 192.

Let

$$\mathcal{R} := \left\{ \left(\mathtt{c}_1, \begin{bmatrix} 0 & \square & 1 & \square & 2 \\ 0 & 0 & \square & \square & \bar{5} \end{bmatrix}, \mathtt{c}_1 \mathtt{c}_1 \mathtt{c}_2 \right), \left(\mathtt{c}_1, \begin{bmatrix} 0 \\ \bar{5} \end{bmatrix}, \mathtt{c}_2 \right) \right\}.$$

Algorithm RH produces, for $\alpha = 64$, the phrase

A fourth example

We work with the A Hirajoshi scale with a tempo of 192.

Let

$$\mathcal{R} := \left\{ \left(\mathtt{c}_1, \begin{bmatrix} 0 & \square & 1 & \square & 2 \\ 0 & 0 & \square & \square & \bar{5} \end{bmatrix}, \mathtt{c}_1\mathtt{c}_1\mathtt{c}_2 \right), \left(\mathtt{c}_1, \begin{bmatrix} 0 & \square \\ \square & 0 \end{bmatrix}, \mathtt{c}_1 \right) \right\}.$$

Algorithm RH produces, for $\alpha = 64$, the phrase

We work with the C major scale with a tempo of 192.

Let

$$\mathcal{R} := \left\{ \left(c_1, \begin{bmatrix} 0 & \square \\ \square & 0 \end{bmatrix}, c_1\right), \left(c_3, \begin{bmatrix} \bar{2} \\ \bar{2} \end{bmatrix}, c_1\right), \left(c_1, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, c_1 c_2\right), \left(c_1, \begin{bmatrix} \square & 2 & \square & 3 \\ 0 & \square & 1 & \square \end{bmatrix}, c_3 c_3\right) \right\}.$$

Algorithm RH produces, for $\alpha = 128$, the phrase

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