PEPS-C1P meeting
13/09/2012 - Paris

Linearity and contiguity, a generalization of the C1P property

Philippe Gambette
LIGM
Université Paris-Est Marne-la-Vallée

Outline

- Contiguity and linearity
- Basic properties
- Links with other graph classes
- Bounds

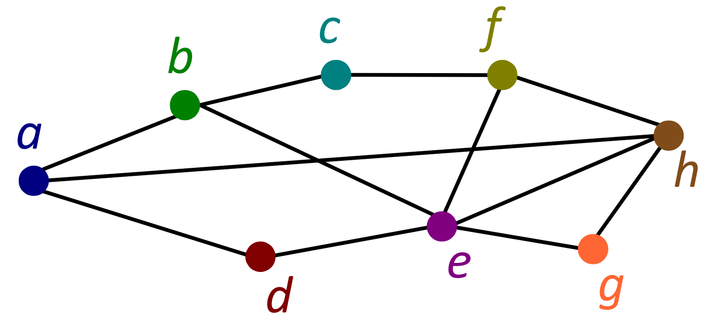
Contiguity & linearity

Closed Contiguity:

$$cc(G) = \min_{\sigma \in S_n} \{ cc(G,\sigma) \}$$

$$cc(G,\sigma) = \max_{x \in G} \{ cc_{G,\sigma}(x) \}$$

$$cc_{G,\sigma}(x) = \min_{\substack{P(x)=\{I_j \text{ intervals of } \sigma\} \\ N[x]=\bigcup_{I_j \in P(x)} I_j}} \{|P(x)|\}$$

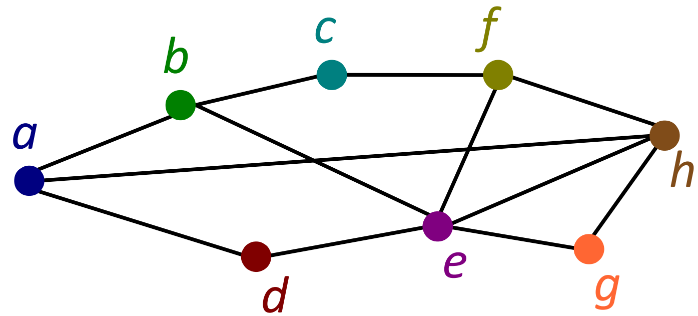


$N[a]$:	a d b c e f h g	$cc_{G,\sigma}(a) = 2$
$N[b]$:	a d b c e f h g	$cc_{G,\sigma}(b) = 2$
$N[c]$:	a d b c e f h g	$cc_{G,\sigma}(c) = 2$
$N[d]$:	a d b c e f h g	$cc_{G,\sigma}(d) = 2$
$N[e]$:	a d b c e f h g	$cc_{G,\sigma}(e) = 2$
$N[f]$:	a d b c e f h g	$cc_{G,\sigma}(f) = 1$
$N[g]$:	a d b c e f h g	$cc_{G,\sigma}(g) = 2$
$N[h]$:	a d b c e f h g	$cc_{G,\sigma}(h) = 2$

Contiguity & linearity

Open Contiguity:

$$\mathrm{oc}(G,\sigma) = \max_{x \in G} \{ \mathrm{oc}_{G,\sigma}(x) \}$$



$N(a)$:	a d b c e f h g	$\mathrm{oc}_{G,\sigma}(a) = 2$
$N(b)$:	a d b c e f h g	$\mathrm{oc}_{G,\sigma}(b) = 2$
$N(c)$:	a d b c e f h g	$\mathrm{oc}_{G,\sigma}(c) = 2$
$N(d)$:	a d b c e f h g	$\mathrm{oc}_{G,\sigma}(d) = 2$
$N(e)$:	a d b c e f h g	$\mathrm{oc}_{G,\sigma}(e) = 2$
$N(f)$:	a d b c e f h g	$\mathrm{oc}_{G,\sigma}(f) = 2$
$N(g)$:	a d b c e f h g	$\mathrm{oc}_{G,\sigma}(g) = 2$
$N(h)$:	a d b c e f h g	$\mathrm{oc}_{G,\sigma}(h) = 3$

Basic properties

For any graph G, $\mathrm{cl}(G) \leq \mathrm{ol}(G)+1 \leq \mathrm{oc}(G)+1 \leq \mathrm{cc}(G)+2$

Complement: For any graph G, $\mathrm{cc}(\overline{G}) \leq \mathrm{cc}(G)+1$

Substitution-composition: For any graphs G and H,

$$\mathrm{cc}(G_{x\leftarrow H}) \leq \max(\mathrm{cc}(G), \mathrm{cc}(H)) + 1$$

Possibly, the neighborhood interval of x containing x is broken by the substitution composition

$$\mathrm{oc}(G_{x\leftarrow H}) \leq \max(\mathrm{oc}(G), \mathrm{oc}(H)) + 1$$

$$\mathrm{cl}(G_{x\leftarrow H}) \leq \max(\mathrm{cl}(G), \mathrm{cl}(H)) + 1$$

(add x & friends in the end of each line) to realize their own closed neighborhood + one line to include them in the neighborhood of all their neighbors in G-H

$$\mathrm{ol}(G_{x\leftarrow H}) \leq \max(\mathrm{ol}(G), \mathrm{ol}(H)) + 1$$

Link with other graph classes

cc(G) = 1 $\Leftrightarrow$ G unit interval graph

oc(G) = 1 $\Leftrightarrow$ G biconvex graph

Given a fixed k, cc(G) = k ? oc(G) = k ? NP-complete

Wang, Lau & Zhao, *DAM*, 2007

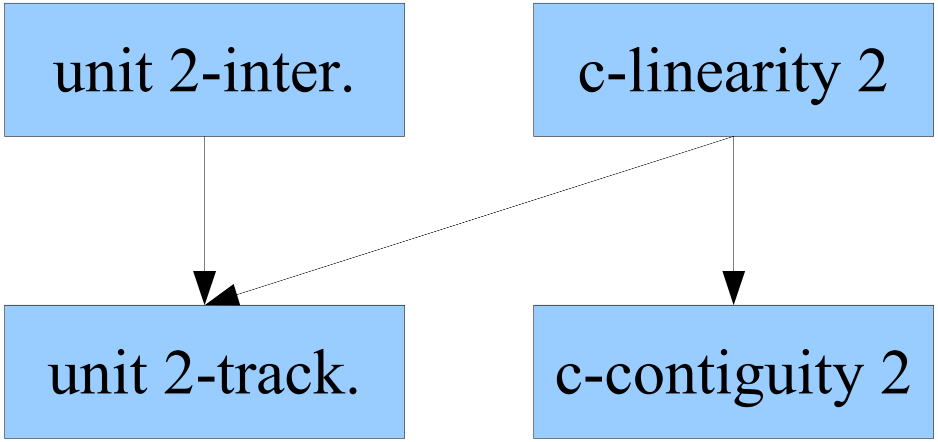
For any graph G, cc(G) $\leq n/4+O(\sqrt{n \log n})$

Gavoille & Peleg, *SIAM JoDM*, 1999

Link with other graph classes

G unit *k*-track graph => cl(*G*) ≤ *k*

Proof : each track coded by one order



Bounds for contiguity

Lower bound for interval graphs and permutation graphs:

There is a family of interval graphs and permutation graphs with *n* vertices having contiguity at least O(log(*n*))

Crespelle & Gambette, 2009

Bounds for cographs:

Every cograph has contiguity at most O(log(*n*)).

There is a family of cographs with *n* vertices having contiguity at least O(log(*n*))

Crespelle & Gambette, 2009

Approximation algorithm for cographs:

There is a constant factor approximation algorithm (approx. ratio 23) to compute the contiguity of cographs .

Crespelle & Gambette, 2012

Tightness of the bounds

Closed contiguity:

The algorithm provides the exact contiguity for any cograph *G* with a binary complete cotree with *n* vertices: $cc(G)=\log(n)/2+1$

Oreste Manoussakis, 2012