Shellable complexes and topology of diagonal arrangements

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Outline

1. Simplicial complexes and diagonal arrangements
2. Some known special cases
3. Main theorem - Homotopy type of L_Δ for shellable Δ
4. $K(\pi, 1)$ examples from matroids

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3. Main theorem - Homotopy type of L_Δ for shellable Δ

4. $K(\pi, 1)$ examples from matroids

Simplicial complexes and diagonal arrangements

A simplicial complex Δ on $[n]$ $\Longleftrightarrow$ a diagonal arrangement $\mathcal{A}_\Delta$: collection of diagonal subspaces $\{x_{i_1} = \cdots = x_{i_k}\}$ of $\mathbb{R}^n$ for all $\{i_1, \ldots, i_k\}$ complementary to facets of Δ

Example

Δ

$\{x_4 = x_5\}$ F_1

$\{x_1 = x_5\}$ F_2

$\{x_1 = x_2 = x_4\}$ F_3

$\{x_1 = x_2 = x_3\}$ F_4

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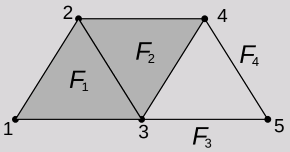

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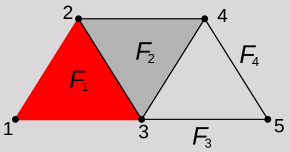

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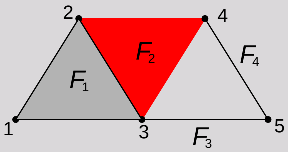

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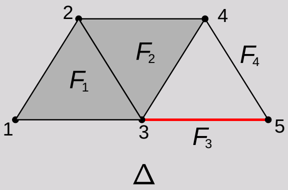

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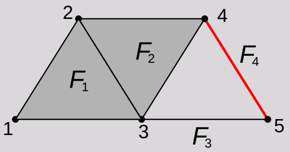

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Simplicial complexes and diagonal arrangements

Example

The Braid arrangement $\mathcal{B}_n = \bigcup_{i<j} \{x_i = x_j\}$

$$\Updownarrow$$

$$\Delta_{n,n-2} = \{\sigma \subset [n] : |\sigma| \leq n-2\}$$

Example

The k-equal arrangement $\mathcal{A}_{n,k} = \bigcup_{i_1<\cdots<i_k} \{x_{i_1} = \cdots = x_{i_k}\}$

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Two important spaces associated with $\mathcal{A}$

Definition

- The complement of an arrangement $\mathcal{A}$ in $\mathbb{R}^n$ is

$$\mathcal{M}_{\mathcal{A}} = \mathbb{R}^n - \bigcup_{H \in \mathcal{A}} H$$

- The singularity link of a central arrangement $\mathcal{A}$ in $\mathbb{R}^n$ is

$$\mathcal{V}^{\circ}_{\mathcal{A}} = \mathbb{S}^{n-1} \cap \bigcup_{H \in \mathcal{A}} H$$

Fact

By Alexander duality,

$$H^i(\mathcal{M}_{\mathcal{A}}; \mathbb{F}) = H_{n-2-i}(\mathcal{V}^{\circ}_{\mathcal{A}}; \mathbb{F})$$

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Application in group cohomology

Definition

An Eilenberg-MacLane space (or a $K(\pi, n)$ space) is a connected cell complex with all homotopy groups except the n-th homotopy group being trivial and the n-th homotopy group isomorphic to π.

Fact

If the CW complex X is a $K(\pi, 1)$ space, then

$$\mathrm{Tor}_n^{\mathbb{Z}\pi}(\mathbb{Z}, \mathbb{Z}) = H_n(X; \mathbb{Z}) \text{ and } \mathrm{Ext}^n_{\mathbb{Z}\pi}(\mathbb{Z}, \mathbb{Z}) = H^n(X; \mathbb{Z}).$$

Theorem (Fadell - Neuwirth, 1962)

Let $\mathcal{B}_n$ be the braid arrangement in $\mathbb{C}^n$. Then $\mathcal{M}_{\mathcal{B}_n}$ is a $K(\pi, 1)$ space.

Theorem (Khovanov, 1996)

Let $\mathcal{A}_{n,3}$ be the 3-equal arrangement in $\mathbb{R}^n$. Then $\mathcal{M}_{\mathcal{A}_{n,k}}$ is a $K(\pi, 1)$ space.

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What is the topology of $\mathcal{M}_{\mathcal{A}}$ and $\mathcal{V}_{\mathcal{A}}^{\circ}$?

Definition

The intersection lattice $L_{\mathcal{A}}$ of a subspace arrangement $\mathcal{A}$ is the collection of all nonempty intersections of subspaces of $\mathcal{A}$ ordered by reverse inclusion.

Theorem (Goresky - Macpherson, 1988)

Let $\mathcal{A}$ be a subspace arrangement in $\mathbb{R}^n$. Then

$$\widetilde{H}^i(\mathcal{M}_{\mathcal{A}}) \cong \bigoplus_{x \in L_{\mathcal{A}} - \{\hat{0}\}} \widetilde{H}_{codim(x)-2-i}(\hat{0}, x).$$

Theorem (Ziegler - Živaljević, 1993)

For every central subspace arrangement $\mathcal{A}$ in $\mathbb{R}^n$,

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What is a general sufficient condition for the intersection lattice $L_{\mathcal{A}}$ of a diagonal arrangement $\mathcal{A}$ to be well-behaved?

Theorem (Björner - Welker, 1995)

The intersection lattice $L_{\mathcal{A}_{n,k}}$ for the k-equal arrangement $\mathcal{A}_{n,k}$ has the homotopy type of a wedge of spheres.

$\mathcal{A}_{n,k} = \mathcal{A}_{\Delta_{n,n-k}}$, and $\Delta_{n,n-k}$ is shellable.

Theorem (Kozlov, 1999)

Let Δ be a simplicial complex on $[n]$ that satisfies some conditions. Then the intersection lattice for $\mathcal{A}_{\Delta}$ is EL-shellable, and hence has the homotopy type of a wedge of spheres.

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Main theorem

Theorem (K.)

Let Δ be a shellable simplicial complex with $\dim \Delta \leq n - 3$. Then the intersection lattice L_Δ of $\mathcal{A}_\Delta$ is homotopy equivalent to a wedge of spheres.

Main theorem (precise version)

Theorem (K.)

Let Δ *be a shellable simplicial complex on* $[n]$ *with* $\dim \Delta \leq n - 3$. *Let* σ *be the intersection of all facets and* $\bar{\sigma}$ *its complement. Then the intersection lattice* L_Δ *is homotopy equivalent to a wedge of spheres, consisting of* $(p-1)!$ *copies of spheres of dimension*

$$\delta(D) = p(2-n) + \sum_{j=1}^{p} |F_{i_j}| + |\bar{\sigma}| - 3$$

for each (unordered) shelling-trapped decomposition $D = \{(\bar{\sigma}_1, F_{i_1}), \ldots, (\bar{\sigma}_p, F_{i_p})\}$ *of* $\bar{\sigma}$.

Moreover, if one removes the $\delta(D)$*-simplex corresponding to a saturated chain* $\overline{C}_{D,\omega}$ *for each shelling-trapped decomposition* $D = \{(\bar{\sigma}_1, F_{i_1}), \ldots, (\bar{\sigma}_p, F_{i_p})\}$ *of* $\bar{\sigma}$ *and a permutation* ω *of* $[p-1]$, *then the remaining simplicial complex* $\widehat{L_\Delta}$ *is contractible.*

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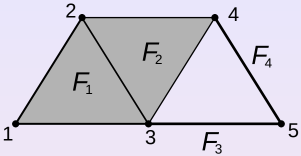
Example



shellable complex Δ

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diagonal arrangement $\mathcal{A}_\Delta$

12345

1234 1235 123/45 1245

123 124 145

15 45

R^5

intersection lattice L_Δ of $\mathcal{A}_\Delta$

order complex of $\overline{L}_\Delta$

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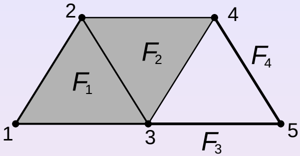

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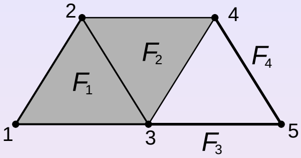

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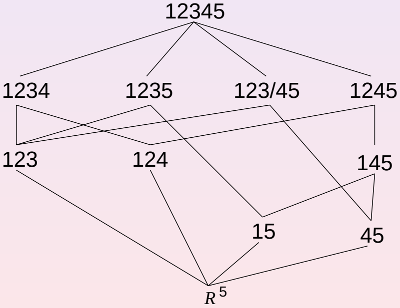
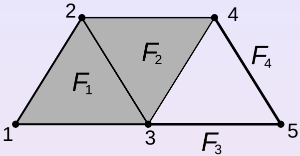
diagonal arrangement $\mathcal{A}_\Delta$



intersection lattice L_Δ of $\mathcal{A}_\Delta$

order complex of $\overline{L}_\Delta$

Example



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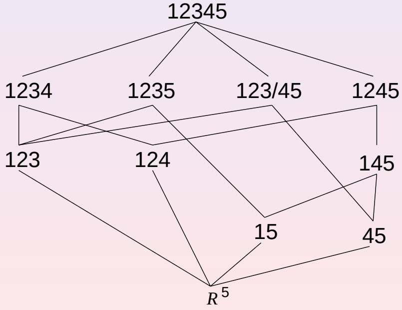

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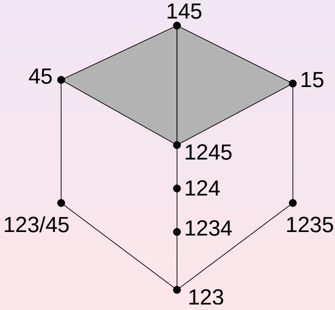

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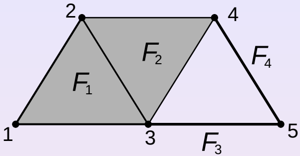

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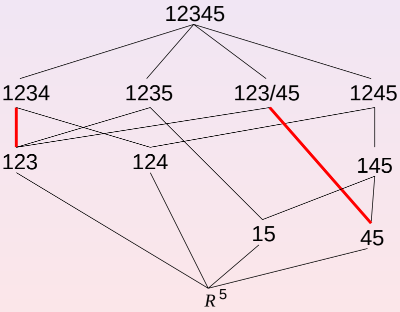

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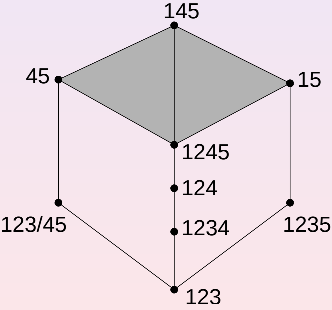

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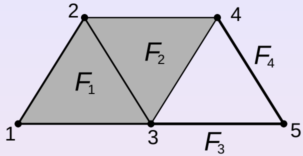

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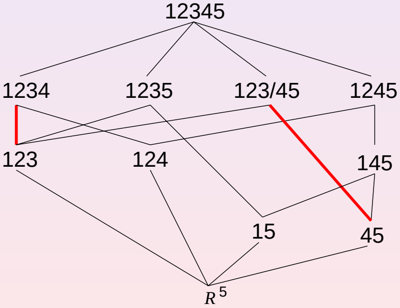

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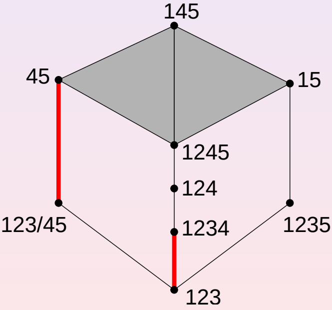

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The topology of $\mathcal{V}^\circ_{\mathcal{A}_\Delta}$

Corollary (K.)

Let Δ be a shellable simplicial complex with $\dim \Delta \leq n-3$. The singularity link of $\mathcal{A}_\Delta$ has the homotopy type of a wedge of spheres, consisting of $p!$ spheres of dimension $n + p(2-n) + \sum_{j=1}^{p} |F_{i_j}| - 2$ for each shelling-trapped decomposition $\{(\bar{\sigma}_1, F_{i_1}), \ldots, (\bar{\sigma}_p, F_{i_p})\}$ of $[n]$.

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Let Δ be a shellable simplicial complex with $\dim \Delta \leq n-3$. Then $\dim_{\mathbb{F}} H_i(\mathcal{V}^\circ_{\mathcal{A}_\Delta}; \mathbb{F})$ is the number of ordered shelling-trapped decompositions $((\bar{\sigma}_1, F_{i_1}), \ldots, (\bar{\sigma}_p, F_{i_p}))$ of $[n]$ with $i = n + p(2-n) + \sum_{j=1}^{p} |F_{i_j}| - 2$.

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Proof sketch of main theorem

Lemma

For the upper interval, there is a simplicial complex whose intersection lattice is isomorphic to $[U_{\bar{\sigma}}, \hat{1}]$. If F is the last facet in the shelling order, the simplicial complex which corresponds to $[U_{\overline{F}}, \hat{1}]$ is shellable.

Proof sketch of main theorem

If F is the last facet in the shelling of Δ, one can consider the following decomposition of $\widehat{\Delta}(\overline{L})$:

$$\widehat{\Delta}(\overline{L}) = \widehat{\Delta}(\overline{L} - \{H\}) \cup \widehat{\Delta}(\overline{L}_{\geq H}),$$

where $\widehat{\Delta}(\overline{L} - \{H\})$ is obtained by removing all chains $\overline{C}_{D,\omega}$ not containing H from $\overline{L} - \{H\}$ and $\widehat{\Delta}(\overline{L}_{\geq H})$ is obtained by removing $\overline{C}_{D,\omega}$ and $\overline{C}_{D,\omega} - H$ from $\overline{L}_{\geq H}$ for all $\overline{C}_{D,\omega}$ containing H. Then one can show that all three spaces $\widehat{\Delta}(\overline{L} - \{H\})$, $\widehat{\Delta}(\overline{L}_{\geq H})$ and their intersection are contractible, and hence $\widehat{\Delta}(\overline{L})$ is also contractible.

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Diagonal arrangement $\mathcal{A}$ such that $\mathcal{M}_\mathcal{A}$ is $K(\pi, 1)$

Theorem (Davis, Januszkiewicz and Scott, 1998)

Let $\mathcal{H}$ be a simplicial real hyperplane arrangement in $\mathbb{R}^n$. Let $\mathcal{A}$ be any arrangement of codimension-2 intersection subspaces in $\mathcal{H}$ which intersects every chamber in a codimension-2 subcomplex. Then $\mathcal{M}_\mathcal{A}$ is $K(\pi, 1)$.

Corollary (K.)

Let $\mathcal{A}$ be a subarrangement of 3-equal arrangement of $\mathbb{R}^n$ so that

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for some collection $T_\mathcal{A}$ of 3-element subsets of $[n]$. Then $\mathcal{A}$ satisfies the hypothesis of DJS's theorem (and hence $\mathcal{M}_\mathcal{A}$ is $K(\pi, 1)$) if and only if every permutation ω in $\mathfrak{S}_n$ has at least one triple in $T_\mathcal{A}$ consecutive.

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DJS matroids

The matroid complexes $\Delta = \mathcal{I}(M)$ are a natural class of shellable complexes.

Definition

Say a rank 3 matroid M on $[n]$ is DJS if its bases $\mathcal{B}(M)$ satisfies the condition of Corollary, i.e., every permutation ω in $\mathfrak{S}_n$ has at least one triple in $\mathcal{B}(M)$ consecutive.

Rank 3 matroids are not always DJS in general.

Proposition (K.)

Let M be a rank 3 matroid on the ground set $[n]$ with no circuits of size 3. Let $P_1, \ldots, P_k$ be distinct parallel classes which have more than one element and let N be the set of all elements which are not parallel with anything else. Then, M is DJS if and only if $\lfloor \frac{|P_1|}{2} \rfloor + \cdots + \lfloor \frac{|P_k|}{2} \rfloor - k < |N| - 2$.

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Theorem (Klivans)

Let M be a matroid whose independent set complex is shifted. Then its bases $\mathcal{B}(M)$ is the principal order ideal of Gale ordering.

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A coordinate subspace arrangement $\mathcal{A}^c_\Delta$ is a collection of coordinate subspaces $\{x_{i_1} = \cdots = x_{i_k} = 0\}$ of $\mathbb{R}^n$ for all $\{i_1, \ldots, i_k\}$ complementary to facets of Δ.

A singularity link of a coordinate subspace arrangement for a shellable complex is homotopy equivalent to a wedge of spheres.

Conjecture (Welker)

A complement of a coordinate subspace arrangement for a shellable complex is homotopy equivalent to a wedge of spheres.

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Characterize the rank 3 matroids which are DJS.

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