

1. Motivation: There are pairs of Grothendieck groups of towers of algebras corresponding to these dual pairs of Hopf algebras. What were the right conditions to impose on a tower of algebras to get the desired algebraic structure on their Grothendieck groups?

4. Axioms for a Tower of Algebras

Why these conditions?

5. Definitions of Multiplications, Comultiplications, Units and Counits on  $G_0(A)$  and  $K_0(A)$ , respectively; a Pairing on  $K_0(A) \times G_0(A)$ ; Result 1

2. Definition of Grothendieck Groups

6. Modification of Axiom (2)(a) and the Definitions of Inductions and Restrictions; Result 2

3. Definition of Hopf Algebra