Formal Power Series and Algebraic Combinatorics
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Hopf algebroids: can combinatorialists help?

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In this talk I shall attempt to explain certain algebraic concepts which seem ripe for combinatorial modelling. I shall avoid technical details by focusing on the underlying ideas, and will concentrate on a few basic examples that I hope convey the flavour of the challenge to those who may not be algebraic experts. I shall certainly not presuppose any familiarity with algebraic topology!

The study of Hopf algebras was initiated by algebraic topologists in the 1930s, and has been permeating other areas of mathematics and theoretical physics ever since. Thanks to the vision of Gian-Carlo Rota and his associates, the theory entered combinatorics during the 1960s, and their viewpoint has now begun to enjoy modest feedback into topology. During the 1970s, however, topologists had already discovered that certain generalisations of Hopf algebras arise rather naturally in stable homotopy theory, and the resulting structures came of age when their status as cogroupoid objects was properly understood.

I shall describe these ideas in terms of examples of two main types. First are those which are particularly straightforward, and therefore illustrate the basic principles rather well to a general audience, and second are those which are relevant to the study of formal power series, and have close ties with algebraic topology. In some of these cases the structures in question are merely Hopf algebras; in others, the full power of algebroids is required. It would be exciting for topologists if combinatorial models could be constructed in these situations, and I shall outline a couple of situations where some success has been achieved in this direction.

Topics I hope to mention include partitions of finite sets, composition of formal power series, and groupoids as graphs.

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