Structure relation and raising/lowering operators for orthogonal polynomials

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The structure relation for classical orthogonal polynomials (OP's), is traditionally defined as a fixed polynomial times the derivative of the n-th degree OP being equal to some explicit linear combination of the OP's of degree n-1, n and n+1, with coefficients depending on n. By substitution of the three-term recurrence relation, the structure relation gives rise to a relation with a raising of lowering operator. A variant of the structure relation can be obtained, for all OP's in the Askey scheme and the q-Askey scheme, by taking the commutator of the second order operator having the OP's as eigenfunctions and the operator of multiplication by x. The lecture will survey past approaches and results on structure relations etc. for OP's in the (q-)Askey scheme and for multivariable OP's associated with root systems. The so-called string equation also pops up here. Then some new results, in particular in the multivariable case will be presented.

Some references:

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Another characterization of the classical orthogonal polynomials,
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(4) T.H. Koornwinder The structure relation for Askey-Wilson polynomials,
arXiv:math.CA/0601303.

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