Crystal bases for quantum generalized Kac-Moody algebras

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We develop the crystal basis theory for quantum generalized Kac-Moody algebras. We define the notion of crystal bases for $U_q(\mathfrak{g})$-modules in the category $\mathcal{O}_{int}$ and prove the standard properties of crystal bases including the tensor product rule. We then prove that there exist crystal bases (and global bases) for $V(\lambda)$ ($\lambda \in P^+$) and $U_q^-(\mathfrak{g})$.

We also introduce the notion of abstract crystals for quantum generalized Kac-Moody algebras and study their fundamental properties. Finally, we prove the crystal embedding theorem and give a characterization of the crystals $B(\infty)$ and $B(\lambda)$.

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