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On bar partitions and spin character zeros

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Abstract. The main combinatorial result in this article is a classification of bar partitions of n which are of maximal p-bar weight for all odd primes $p \leq n$. As a consequence, we show that apart from very few exceptions any irreducible spin character of the double covers of the symmetric and alternating groups vanishes on some element of odd prime order.

Résumé. Notre résultat principal est une classification des partages barrés de n qui ont un poids p-barré maximal pour tous les nombres prémiers p impairs inférieur à n. Comme conséquence, on a que, à quelques exceptions près, tout caractère spin irréductible d'une couverture double des groupes symètriques et groupes alternants s'annule sur un élément d'ordre premier.

1. Introduction

A well known result by Burnside states that any non-linear irreducible character of a finite group vanishes on some element of the group. This was refined in [**9**], where it was shown that such a character always has a zero at an element of prime power order; it had also been noticed in [**9**] that any non-linear irreducible character of a finite simple group except possibly the alternating groups even vanishes on some element of prime order. This was complemented in [**5**] where it was shown that this character property also holds for the symmetric and the alternating groups. Indeed, this vanishing property was a consequence of a combinatorial result on the weights of partitions.

Here, we deal with the corresponding result on bar weights of partitions into distinct parts (which we call bar partitions). This then yields a vanishing property for irreducible spin characters of the double covers of the symmetric and alternating groups on elements of odd prime order.

In the next section we collect together some combinatorial preliminaries; we then briefly recall the results from [**5**] in the case of partitions and ordinary characters of the symmetric and alternating groups. In Section 4 we discuss the case of bar partitions and spin characters of the double cover groups; in the main result, Theorem 4.1, the bar partitions of n are classified which are of maximal p-bar weight for all primes $p \leq n$. These then give rise to the desired spin character zeros; see Theorem 4.2.

A detailed paper with full proofs will appear elsewhere.

2. Preliminaries

We refer to [**8**], [**12**], [**7**] for details about partitions, Young diagrams, hooks and bar partitions, shifted diagrams and bars, respectively.

Consider a partition $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_l)$ of the integer n. Thus $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_l > 0$ and $\lambda_1 + \lambda_2 + \ldots + \lambda_l = n$, with integer parts λ_i; $l = l(\lambda)$ is the length of λ. The Young diagram of λ consists of n boxes with λ_i boxes in the ith row. We refer to the boxes in matrix notation, i.e. the (i,j)-box is the

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jth box in the ith row. The (i,j)-hook consists of the boxes in the Young diagram to the right of and below the (i,j)-box, and including this box. The number of boxes in this hook is its hook length, denoted by h_{ij}.

For $n \in \mathbb{N}$, we denote by $D(n)$ the set of partitions of n into distinct parts, and we set $D = \bigcup_n D(n)$. We call the partitions in D also bar partitions. A partition $\lambda \in D(n)$ is in $D^+(n)$ (or $D^-(n)$, respectively) if $n - l(\lambda)$ is even (or odd, respectively).
We denote by $\mathcal{O}(n)$ the set of partitions of n into odd parts; elements of the double cover groups $\tilde{S}_n$ which correspond to elements of S_n of cycle type $\alpha \in \mathcal{O}$ are said to be of type $\mathcal{O}$.
For $\lambda \in D$, we consider the corresponding shifted diagram, where in the ith row we start on the diagonal at (i,i) rather than at the box $(i,1)$. By flipping over the diagonal we obtain the shift symmetric diagram $S(\lambda)$. The bar lengths in λ correspond to the hook lengths in the λ-boxes of $S(\lambda)$; the bar length at position (i,j) is then denoted b_{ij}; we abbreviate the bar lengths in the first row by $b_{1i} = b_i$.

Example. Take $\lambda = (4,3,1)$. In the shift symmetric diagram below the bar lengths are filled into the corresponding boxes of λ.

$$\begin{array}{ccccc} . & 7 & 5 & 4 & 2 \\ . & . & 4 & 3 & 1 \\ . & . & . & 1 & \\ . & . & & & \end{array}$$

The removal of a p-bar from $\lambda \in D(n)$ corresponds to taking a part p or two parts summing to p out of λ, or subtracting p from a part of λ if possible (i.e., if the resulting partition is in $D(n-p)$). Doing this as long as possible gives the $\bar{p}$-core $\lambda_{(\bar{p})}$ of λ; the number of p-bars removed is then the p-bar weight $\bar{w}_p(\lambda)$ of λ (see [**7**] or [**12**] for details). These operations may also be performed on a suitable $\bar{p}$-abacus.

Example. Take $p=3$, $\lambda = (7,3,2,1)$. Removing a bar of length 3 from λ can be achieved by removing the parts 2 and 1 from λ, or by removing the part 3, or by replacing 7 by 4. When we do this in succession, we have reached the bar partition (4), from which we can remove a further 3-bar and thus obtain $(1) = \lambda_{(\bar{3})}$; the $\bar{3}$-weight of λ is thus 4.

We will often make use of the following property of the p-bar weight of a partition (see [**11**], [**12**]); the Lemma may easily be proved by considering the $\bar{p}$-abacus (see [**12**]).

Lemma 2.1. *Let p be an odd prime. If λ is a bar partition of $\bar{p}$-weight $\bar{w}_p(\lambda) = w$, then λ has exactly w bars of length divisible by p. In particular, if λ has a bar of length divisible by p, then it has a bar of length p.*

This is used to prove some easy but crucial results about bar lengths (compare this with [**4**] where a similar Lemma for hook lengths is used).

For $p=2$, a suitable parameter to consider is the $\bar{4}$-core of λ which is computed using the $\bar{4}$-abacus with one runner for the even parts, and two conjugate runners for the parts $\equiv 1,3 \mod 4$; in contrast to the $\bar{p}$-abacus for odd p, here we are allowed to subtract 2 from the even parts (so these will be removed when computing the $\bar{4}$-core).

3. Partitions and ordinary characters of S_n and A_n

Before stating the new results on bar partitions and spin characters in the next section, we recall here the recent results from [**5**]. Towards the refinement of Burnside's Theorem for S_n and A_n the following main combinatorial result was proved there:

Theorem 3.1. [**5**] *Let λ be a partition of $n \in \mathbb{N}$. Then the following holds:*
(i) *λ is of maximal p-weight for all primes $p \le n$, if and only if one of the following occurs:*

$$\lambda = (n)\,,(1^n) \text{ or } (2^2)\,.$$

(ii) *λ is of maximal p-weight for all odd primes $p \leq n$, if and only if λ is one of the partitions in* (i)*, one of* $(n-1,1)$, $(2,1^{n-2})$*, where $n = 2^a + 1$ for some $a \in \mathbb{N}$, or one of the following occurs:*

$$\begin{array}{rrl} n = & 6: & \lambda = (3,2,1) \\ n = & 8: & \lambda = (5,2,1) \text{ or } (3,2,1^3) \\ n = & 9: & \lambda = (6,3) \text{ or } (2^3,1^3) \\ n = & 10: & \lambda = (4,3,2,1) \end{array}$$

This has the desired consequence:

THEOREM 3.2. [**5**] *Let $n \in \mathbb{N}$. Let χ be any non-linear irreducible character of the symmetric group S_n or the alternating group A_n. Then χ vanishes on some element of prime order. If $\chi(1)$ is not a 2-power, then χ is zero on some element of odd prime order.*

Theorem 3.1 also has a consequence for the distribution into p-blocks; this was recently taken up in more detail in [**2**].

We refer to [**8**, section 2.5] for the labelling of the irreducible characters of A_n. A simple relation between the p-weight of a partition λ and the defect of the p-block containing the irreducible character labelled by λ is given in [**8**, 6.2.45]. The principal p-block of a finite group is the block containing the trivial character.

THEOREM 3.3. [**5**] (i) *The characters $[n], [1^n]$ and $[2^2]$ are the only irreducible characters of S_n which are in p-blocks of maximal defect for all primes p.*
Apart from $[1^2], [1^3], [1^4], [1^6], [2^2]$, the trivial character of S_n is the only irreducible character which is in the principal p-block for all primes $p \leq n$.
(ii) *The characters $\{n\}$, $\{2,1\}_\pm$ and $\{2^2\}_\pm$ are the only irreducible characters of A_n which are in p-blocks of maximal defect for all primes p.*
They belong to the principal p-block for all primes $p \leq n$, except for the characters $\{2,1\}_\pm$ at $p = 2$.

We will see that our main result on bar partitions is of a similar type as Theorem 3.1 above, and it has similar consequences for character zeros of spin characters and for the distribution of spin characters into spin p-blocks, for odd primes p.

4. Bar partitions and spin characters

In our main result we present a classification of the bar partitions of n which have maximal $\bar{p}$-weight $\left\lfloor \frac{n}{p} \right\rfloor$ for all odd primes $p \leq n$; equivalently, the $\bar{p}$-core of these bar partitions is small in the sense that it is of size smaller than p. (Here $\lfloor \cdot \rfloor$ denotes the floor function. Thus $\lfloor x \rfloor$ is the integral part of $x \in \mathbb{R}$.) For $p = 2$, we consider the case where the $\bar{4}$-core is small, i.e., of size smaller than 4.

The elements of odd prime order p which we are then going to use for the vanishing property for spin characters of the double cover $\tilde{S}_n$ of the symmetric group S_n are those where the corresponding cycle type is of maximal p-bar weight, i.e., the cycle type has $\left\lfloor \frac{n}{p} \right\rfloor$ parts of size p. Indeed, the connection to the vanishing of spin character values is easily explained. The irreducible spin characters of $\tilde{S}_n$ are labelled by the bar partitions λ of n (and signs). The recursion formula given by Morris [**10**] for spin character values on elements of type $\mathcal{O}$ in $\tilde{S}_n$ shows that the irreducible spin character(s) labelled by λ vanishes on a p-element of maximal weight (where p is odd), if the $\bar{p}$-weight of λ is *not* maximal.

Our main result on bar partitions is the following:

THEOREM 4.1. *Let λ be a bar partition of $n \in \mathbb{N}$. Then λ is of maximal $\bar{p}$-weight for all odd primes $p \leq n$, if and only if $\lambda = (n)$ or $\lambda = (n-1,1)$, where $n = 2^a + 2$ for some $a \in \mathbb{N}$, or one of the following occurs:*

$$\begin{array}{rrl} n = & 5: & \lambda = (3,2) \\ n = & 6: & \lambda = (3,2,1) \\ n = & 8: & \lambda = (5,2,1) \\ n = & 9: & \lambda = (4,3,2) \\ n = & 10: & \lambda = (4,3,2,1) \text{ or } (7,3) \end{array} .$$

If, in addition, also $\lambda_{(\bar{4})}$ is small, then $\lambda = (n)$ or λ is one of $(3,1)$, $(3,2,1)$, $(4,3,2,1)$.

The combinatorial classification result immediately has the desired consequence for the spin character zeros, as explained above; first we have to introduce some more notation (see [**7**], [**10**], [**12**]).

We denote by $\langle\mu\rangle$ the irreducible spin character of $\tilde{S}_n$ corresponding to $\mu \in D^+(n)$, and by $\langle\mu\rangle_+$, $\langle\mu\rangle_- = \mathrm{sgn}\cdot\langle\mu\rangle_+$, the irreducible spin characters of $\tilde{S}_n$ associated to $\mu \in D^-(n)$.
Furthermore, we let $\langle\langle\mu\rangle\rangle$ denote the irreducible spin character of $\tilde{A}_n$ corresponding to $\mu \in D^-(n)$ (which is the reduction of $\langle\mu\rangle_\pm$), and $\langle\langle\mu\rangle\rangle_\pm$ the irreducible spin characters of $\tilde{A}_n$ associated to $\mu \in D^+(n)$ (which are conjugate and sum to the reduction of $\langle\mu\rangle$, and which differ only on classes of cycle type $\lambda \in D^+$).

We refer to [**7**] for further details on the irreducible spin characters of $\tilde{A}_n$.

Let $n \in \mathbb{N}$, $n \geq 4$. First we observe that any irreducible spin character of a double cover $\tilde{S}_n$ of the symmetric group or a double cover $\tilde{A}_n$ of the alternating group has a zero of order 2. For this, note that the cycle types (2^a1^b), with $a > 0$, are neither of type $\mathcal{O}$ nor of type D and hence these classes do not split in the double cover groups. Thus all spin characters are zero on these classes. Hence in the following Theorem we are only interested in classes of odd prime order.

Theorem 4.2. *Let $n \in \mathbb{N}$, $n \geq 4$. Let χ be any irreducible spin character of a double cover of the symmetric group $\tilde{S}_n$ or the alternating group $\tilde{A}_n$. Then χ vanishes on some element of odd prime order, except if χ is a basic spin character, i.e., labelled by (n), or in the cases where χ is labelled by $(n-1,1)$ with $n = 2^a + 2$ for some $a \in \mathbb{N}$, or by one of the partitions $(3,2)$, $(3,2,1)$ or $(5,2,1)$.*

Remark 4.3. If an irreducible character χ of a finite group G has a zero at an element of prime order p, then p divides $\chi(1)$. Note that the irreducible spin characters of $\tilde{S}_n$ and $\tilde{A}_n$ of prime power degree have been classified in [**1**]; from Theorem 4.2 we can immediately recover the classification of irreducible spin characters of 2-power degree for these groups. In fact, here they are exactly those that do not have a zero at an element of odd prime order.
The converse of the statement above does not hold, even for $G = \tilde{S}_n$. The spin character $\langle 8,4\rangle$ is of degree $5280 = 2^4\cdot 3\cdot 5\cdot 11$, but the character does not vanish on any element of order 3.

Note that for $p > 2$ there is a simple relation between the $\bar{p}$-weight of a bar partition λ and the defect of the p-spin block containing the irreducible spin character(s) of S_n or A_n labelled by λ (see [**12**]). For $2 < p \leq n$, the basic spin character(s) of $\tilde{S}_n$ or $\tilde{A}_n$ are contained in one spin p-block which we call the *basic spin p-block* of $\tilde{S}_n$ or $\tilde{A}_n$, respectively. The following is then another direct consequence of Theorem 4.1 (note that for $a > 2$ the spin character to $(2^a+1,1)$ is not in the basic spin p-block for any odd prime p not dividing n and $n-1$).

Theorem 4.4. *Let $n \in \mathbb{N}$, $n \geq 4$.*
(i) *The basic spin characters $\langle n\rangle_{(\pm)}$, the spin characters $\langle n-1,1\rangle_{(\pm)}$ where $n = 2^a+2$ for some $a \in \mathbb{N}$, and the spin characters $\langle 3,2\rangle_\pm, \langle 3,2,1\rangle_\pm, \langle 5,2,1\rangle_\pm, \langle 4,3,2\rangle, \langle 4,3,2,1\rangle, \langle 7,3\rangle$ are the only irreducible spin characters of $\tilde{S}_n$ which are in spin p-blocks of maximal defect for all odd primes p.*
The spin characters $\langle 3,1\rangle$, $\langle 5,1\rangle$, $\langle 3,2\rangle_\pm$, $\langle 3,2,1\rangle_\pm$, $\langle 4,3,2\rangle$, $\langle 7,3\rangle$ are the only non-basic spin characters contained in the basic spin p-block for all odd primes $p \leq n$.
(ii) *The basic spin characters $\langle\langle n\rangle\rangle_{(\pm)}$, the spin characters $\langle\langle n-1,1\rangle\rangle_{(\pm)}$ where $n = 2^a+2$ for some $a \in \mathbb{N}$, and the spin characters $\langle\langle 3,2\rangle\rangle$, $\langle\langle 3,2,1\rangle\rangle$, $\langle\langle 5,2,1\rangle\rangle$, $\langle\langle 4,3,2\rangle\rangle_\pm$, $\langle\langle 4,3,2,1\rangle\rangle_\pm$, $\langle\langle 7,3\rangle\rangle_\pm$ are the only irreducible spin characters of $\tilde{A}_n$ which are in spin p-blocks of maximal defect for all odd primes p.*
The spin characters $\langle\langle 3,1\rangle\rangle_\pm, \langle\langle 5,1\rangle\rangle_\pm, \langle\langle 3,2\rangle\rangle, \langle\langle 3,2,1\rangle\rangle, \langle\langle 4,3,2\rangle\rangle_\pm, \langle\langle 7,3\rangle\rangle_\pm$ are the only non-basic spin characters contained in the basic spin p-block of $\tilde{A}_n$ for all odd primes $p \leq n$.

For $p = 2$, the blocks contain both ordinary and spin characters; in fact, the 2-block distribution of spin characters is more intricate and has been determined in [**3**]. Here the $\bar{4}$-combinatorics mentioned before fits with the distribution of spin characters into the 2-blocks of $\tilde{S}_n$ (see [**3**]). We note that when $n \equiv 3 \mod 4$, the basic spin character is not contained in the principal 2-block. Using also the 2-blocks, the non-basic spin characters may be even more finely separated from the basic spin characters; one easily checks that only the spin characters $\langle 3,1\rangle$ and $\langle 3,2,1\rangle_\pm$ are in the same p-block as the basic spin characters for all primes $p \leq n$ (analogously for $\tilde{A}_n$).

Now we want to indicate the strategy of the proof of the main classification result. We start by studying the bar lengths in bar partitions. We write $\bar{h}_\mu$ for the product of all the bar lengths of a bar partition μ.

From now on, $\lambda = (\lambda_1, \dots, \lambda_l)$ is always a bar partition of n, of length l. The following easy result is very useful:

PROPOSITION 4.5. *Assume that $\bar{w}_p(\lambda) = \left\lfloor \frac{n}{p} \right\rfloor$ for the odd prime $p \le n$.*

(i) *Let μ be obtained from λ by removing the first row. If p does not divide $\bar{h}_\mu$, then $p,\ 2p,\ \dots,\ \left\lfloor \frac{n}{p} \right\rfloor p$ are first row bar lengths of λ.*
(ii) *If $n - \lambda_1 < p$, then $p,\ 2p,\ \dots,\ \left\lfloor \frac{n}{p} \right\rfloor p$ are first row bar lengths of λ.*

Note that the first row bar lengths of λ, denoted $b_1, \dots, b_{\lambda_1}$, can explicitly be given; the set of these numbers is

$$\{\lambda_1 + \lambda_2, \dots, \lambda_1 + \lambda_l\} \cup \{1, \dots, \lambda_1\} \setminus \{\lambda_1 - \lambda_2, \dots \lambda_1 - \lambda_l\}\,.$$

In particular, the largest bar length in λ is $\lambda_1 + \lambda_2$.

As for the study of hook lengths of partitions, some number theoretic results about the distribution of primes are needed. In particular, a result due to Hanson is very useful; the exceptions occurring here are also a reason for exceptions occurring for small n in the classification theorem.

THEOREM 4.6. [**6**] *The product of k consecutive numbers all greater than k contains a prime divisor greater than $\frac{3}{2}k$, with the only exceptions $3 \cdot 4$, $8 \cdot 9$ and $6 \cdot 7 \cdot 8 \cdot 9 \cdot 10$.*

In the case of partitions, we first dealt with the case of hooks in [**5**]. Here, one treats the "bar case" first, i.e., partitions of length at most 2.

PROPOSITION 4.7. *Let $\lambda = (n-k, k)$ for some $k \in \mathbb{N}_0$, $k < n-k$. Then $\bar{w}_p(\lambda) = \left\lfloor \frac{n}{p} \right\rfloor$ for all odd primes $p \le n$ if and only if one of the following holds:*

(i) *$k = 0$, i.e., $\lambda = (n)$.*
(ii) *$k = 1$ and $n = 2^a + 2$ for some $a \in \mathbb{N}_0$, i.e., $\lambda = (2^a + 1, 1)$.*
(iii) *λ is one of $(3,2)$, $(7,3)$.*

If, in addition, also the $\bar{4}$-core is small, then $\lambda = (n)$ or λ is one of $(2,1), (3,1)$.

The following observation is crucial for getting a reduction procedure started in the general case.

LEMMA 4.8. *Let $\lambda \in D(n)$. Let s be a bar length of λ with $\frac{n}{2} \le s$. Then s is a first row bar length of λ or $s = b_{23} = \lambda_2 + \lambda_3$. In the second case, b_1, b_2 are then the only first row bar lengths $\ge \frac{n}{2}$.*

COROLLARY 4.9. *Let $n = 13, 14$ or $n \ge 17$. Let $\lambda \in D(n)$ be of maximal $\bar{p}$-weight for all odd primes p with $\frac{n}{2} \le p \le n$. Then all bar lengths $\ge \frac{n}{2}$ are first row bar lengths of λ.*

Based on the following result we can then use the same algorithm as in [**4**]:

PROPOSITION 4.10. *Let $\lambda \in D(n)$, $n \ge 17$, which is of maximal $\bar{p}$-weight for all odd primes $p \le n$. Let $s_1 < s_2 < \cdots < s_r \le n$ and $t_1 < t_2 < \cdots < t_r \le n$ be sequences of integers satisfying*

(i) *$s_i < t_i$ for all i;*
(ii) *s_1, t_1 are primes $> \frac{n}{2}$;*
(iii) *for $1 \le i \le r-1$, s_{i+1}, t_{i+1} have prime divisors exceeding $2n - s_i - t_i$.*

Then $s_1, \dots, s_r$, $t_1, \dots, t_r$ are first row bar lengths of λ.

It was already checked for the proof of the classification result in [**1**] that a suitable algorithm producing sequences as occurring in the proposition above ends close to n; also, the Theorem is easily checked for small n. We then obtain the following consequence:

COROLLARY 4.11. *Let $n \in \mathbb{N}$. Let λ be a bar partition of n of maximal $\bar{p}$-weight for all odd primes $p \le n$, $b_1 = \lambda_1 + \lambda_2$ its largest bar length.*

(i) *For $n \le 9.25 \cdot 10^8$, $n - b_1 \le 4$.*
(ii) *For $n > 9.25 \cdot 10^8$, $n - b_1 \le 225$.*

After this, we still have the tasks to reduce 225 to some manageable number, and to deal with the cases where $n - b_1$ is small. For this, we use a tailor-made number-theoretic Lemma for reducing $d = n - b_1$ and $k = \lambda_2 - \lambda_3 - 1$; it refines Hanson's Theorem in special situations.

LEMMA 4.12. *Let* $5 \leq m \leq 1000$. *Then any product of* m *consecutive integers larger than* $5.5 \cdot 10^8$ *has a prime divisor* $q > 2.15 \cdot m$, *when* $m \leq 10$, $q > 2.58 \cdot m$, *when* $11 \leq m \leq 21$, *and* $q > 3 \cdot m$, *when* $m \geq 22$.

This Lemma also helps to deal with the cases of medium-sized d and k. The cases of small d and k are dealt with in a tedious case-by-case analysis; here the further exceptions for small n stated in the Theorem arise. This then finishes the proof.

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