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Special Issue: Dedicated to the Memory of Marcel-Paul Schützenberger

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## MARCEL-PAUL SCHÜTZENBERGER (1920–1996)

Jean-Éric Pin

Dedicated to the Memory of Marcel-Paul Schützenberger

This special issue of IJAC is dedicated to the memory of Marcel-Paul Schützenberger (1920–1996). Marcel-Paul Schützenberger died on Monday, July 29, 1996.

Schützenberger was born in Paris on October 24, 1920 in an Alsatian family. During the war, he served in the “Forces Françaises Combattantes” from September 1943 to August 1944. He became Doctor in Medicine in 1948 and Doctor in Mathematics in 1953. In the meantime, from 1948 to 1953, he occupied a position at the National Institute of Hygien, where he contributed in particular to the discovery of the gene of trisomy. From 1953 to 1956, he occupied a research position at the C.N.R.S (National Center of Scientific Research). During the academic year 1956–1957, he was a research associate at MIT in the team of C. Shannon. He obtained an academic position at the University of Poitiers in 1957. He was a visiting professor at the University of North Carolina in 1960–1961 and at the Harvard Medical School in 1961–1962. He was back in Poitiers in 1962 before moving to Paris in 1963, first as a research director at the C.N.R.S. and then as a full professor at the University of Sciences in 1964. He also spent the spring of 1963 at the University of Pennsylvania as a research associate and the summer of 1963 as an expert at EURATOM. He became a scientific director at IRIA (nowadays INRIA) from 1968 to 1972 and then spent the next year in Naples, in Italy. He was a consultant at the IBM research center and at the Rand Corporation in the summer of 1962 and 1966 respectively. He was also a member of the scientific direction of the World Health Organization from 1961 to 1980. He was elected a corresponding member of the French Academy of Sciences in 1979 and was promoted to full membership in 1988.

Schützenberger's contributions cover medicine (he received the Larrey prize from the Academy of Medicine in 1969), statistics (he received the Montyon prize of statistics from the Academy of Sciences in 1964), algebra, combinatorics and computer science.

Although he made tremendous contributions to combinatorics, including symmetric functions and Young tableaux, combinatorics on words, bijective proofs, I will just discuss here some of his work on semigroups and theoretical computer science. His interest in the emerging field of computer science goes back to the early fifties.

In 1953, he published a note entitled “Remarques sur un problème de codage binaire”. Then in 1955, his seminal paper founded the theory of variable length codes. Let me quote D. Perrin: “in 1955, he published a paper (*Une théorie algébrique du codage*), presented at the algebra seminar in Paris, which already contains many of the ideas of his work on automata. For example, the definition of the syntactic semigroup, of recognizable sets and their equality with rational sets, was almost simultaneous with the appearance of Kleene’s work. The problem of understanding the nature of the property of unique decipherability fascinated him from the beginning. He found an incredible interplay between algebra through the use of finite semigroups, probability theory and combinatorics. Many of his favourite subjects were put together.” This seminal paper, and his numerous subsequent publications on this field would ultimately develop as an autonomous theory which is presented in the book of J. Berstel and D. Perrin *Theory of Codes* (Academic Press, 1985). But it is worth mentioning that his main conjecture on finite codes, the so-called *factorization conjecture*, is still open.

His contribution to the theory of automata of formal languages is outstanding. He pushed his visionary idea of replacing automata by algebraic structures in several directions. What is nowadays known as the Kleene-Schützenberger theorem is a remarkable extension of Kleene’s theorem to formal power series in noncommutative variables. This point of view also ultimately led to an autonomous theory — see for instance the book of Berstel and Reutenauer *Rational series and their languages* (Springer, 1988). In the same vein, Schützenberger would also view algebraic power series as a generalization of context-free languages and would give a purely algebraic interpretation of relations and transducers on words. He wrote several articles about rational functions and relations, and some of his early results still seem to be periodically rediscovered. He was also the first to understand the deep connection between free groups and context-free languages. His paper *The algebraic theory of context-free languages*, published jointly with N. Chomsky in 1963, is a cornerstone of the theory of formal languages.

According to S. Eilenberg, Schützenberger’s celebrated theorem on star-free languages is, right after Kleene’s theorem, the most important result of the theory of finite automata. This result, published in 1965, states that a language is star-free if and only if its syntactic monoid is finite and aperiodic. It is considered as the starting point of the theory of varieties, formalized later on by S. Eilenberg. Samuel Eilenberg says in the preface of his first volume: “The reader will find that the name of M. P. Schützenberger is often mentioned as author (or coauthor with me) of many of the new results or proofs that appear in this volume; most have not been previously published. However, his contributions went much beyond that; virtually every phase of the development presented here was endlessly discussed with him.” Indeed Schützenberger made several other fundamental contributions to this field. In particular, he gave, in a joint article with S. Eilenberg, the characterization of varieties of semigroups by ultimate identities. He also characterized the unambiguous star-free languages and the languages whose syntactic monoids have only commutative subgroups. He also gave another characterization of the star-free languages using union, product and a restriction of the star operation. All these results had, and actually still have, a strong influence on the development of this theory.

His contribution to the theory of semigroups was certainly influenced by his work on codes and automata theory. For instance, he studied with S. Eilenberg, the rational subsets of a commutative monoid and introduced a new operation on semigroups, nowadays called the Schützenberger product, in his study of the star-free languages. The Schützenberger group and the Schützenberger representation of a  $\mathcal{D}$ -class are also by now standard notions in semigroup theory. Both notions play a prominent role in recent research on semigroups. He also gave, in a joint paper with Coudrain, one of the first nontrivial finiteness conditions for finitely generated monoids. One should also mention his beautiful results relating Lie algebra decompositions with factorizations of the free monoid.

Although I have not been a student of Schützenberger myself, my research was and still is largely inspired by his work. Let me finish with two personal memories I have of him.

The first one takes place during the spring school on the theory of codes in 1979, in Jougue. Schützenberger was taking part in the rock-and-roll dancing party organized by the youngest participants, when, observing at 2 A.M. that the tempo was declining, he offered “vitamins” to everybody. These vitamins, actually a strong pear alcohol, knocked me down to bed in the next half hour. The next day, I was rather impressed to see Schützenberger, who has stayed at least until 3 A.M. at the party, sitting, or rather lying down on the first row of the seminar room in his usual favourite style, and asking the most accurate questions to the speakers of the morning session.

The second one is a conversation I had with him on the telephone, a few months before his death. He was actually calling the secretary’s office for another reason, but when I answered, he immediately started to discuss mathematical issues interspersed with humorous comments about the deficiencies of Darwinism. . . .

Marcel-Paul Schützenberger was a truly pluridisciplinary researcher who had decisive contributions in several branches of Science and even created some of them. His unforgettable personality, which had a strong influence on all his friends and former students, was described by Herbert Wilf as follows: “He was possessed of a lively curiosity, a brilliant mind, a passion for all kinds of mathematics and for substantive intellectual achievement of all sorts, a low tolerance for poseurs and fools, a warm and vibrantly supportive personality as regards his interaction with young people of promise, a deep affection for the human race despite its numerous foibles, and a personal charm, grace and good humor that was totally captivating.”

The name of Marcel-Paul Schützenberger will stay much alive in the memories of his friends.<sup>a</sup>

<sup>a</sup>The reader is invited to visit the following Web sites dedicated to his memory

[http://cs.anu.edu.au/publications/eljc/Volume\\_3/Html/v3i1f1.html](http://cs.anu.edu.au/publications/eljc/Volume_3/Html/v3i1f1.html)

<http://www-igm.univ-mlv.fr/perrin/Recherche/Publications/Marco/>

Another source of information is the volume *Mots* (by Lothaire, Hermes, 1990), a collection of papers dedicated to Schützenberger by his friends and former students. At least two other scientific journals have published obituaries of Schützenberger: *Semigroup Forum* (Vol 55, pp. 135–151, 1997) and *Theoretical Computer Science* (Vol 204, Numbers 1–2, 1998).