Solution to Exercise 4.2.2

We start with the identity to be proven, and transform it by algebraic operations and inversions. The actual proof is obtained by going backwards. Let \( x = 1 - a \) and \( y = b \). We have to prove that

\[
\frac{1}{2}(x + iy)^{-1} + \frac{1}{2}(x - iy)^{-1} = (x + yx^{-1}y)^{-1}.
\]

Multiply both sides by 2, by \( x + yx^{-1}y \) on the left and by \( x + iy \) on the right. This gives

\[
x + yx^{-1}y + (x + yx^{-1}y)(x - iy)^{-1}(x + iy) = 2(x + iy).
\]

Writing \( x + iy = x - iy + 2iy \), and cancelling \( x \) on both sides, we obtain:

\[
yx^{-1}y + x + yx^{-1}y + (x + yx^{-1}y)(x - iy)^{-1}2iy = x + 2iy.
\]

Now cancel \( x \) again and divide by \( y \) on the right (NB: backwards, this will be multiplication by \( y \), so we do not invert noninvertible series, as predicted by Theorem 2.1):

\[
2yx^{-1} + 2i(x + yx^{-1}y)(x - iy)^{-1} = 2i.
\]

Multiply by \( x - iy \) on the right:

\[
2yx^{-1}(x - iy) + 2i(x + yx^{-1}y) = 2i(x - iy).
\]

That is:

\[
2y - 2iyx^{-1}y + 2ix + 2iyx^{-1}y = 2ix + 2y.
\]

Formidable!