Permutation Pattern Matching for Separable Permutations.

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Plan

Permutation.

Permutation

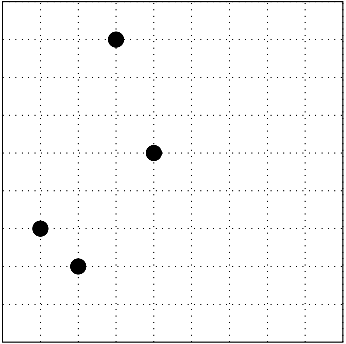
Two orders over a finite set.

- Usually the ordered set is a set of integers.
- Written as word $\pi = \pi[1]\pi[2]\ldots\pi[n]$.

Plot of a Permutation.

- We associate a figure to a permutation called a plot.
- Each element of a permutation is represented by the point $(i, \pi[i])$.

Example : the plot of the permutation 3 2 8 5.



Reduced form of a permutation and reduction.

Reduced Form of a Permutation

The elements are the first n integers.

Obtained by reducing a permutation.

- If π is on the set $\{e_1, e_2, \ldots, e_n\}$ where the naturall order is $e_1 < e_2 < \ldots < e_n$, we obtain the reduced form of π by renaming every e_i by i.

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- 3 2 8 5 becomes 2 1 4 3

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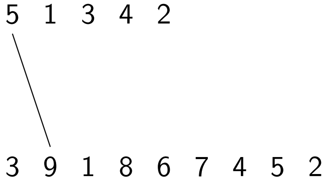
We say that σ occurs in π if such mapping exists.

Example : occurrence of 51342 in 391867452.

The mapping that map :

- 1 to 2,



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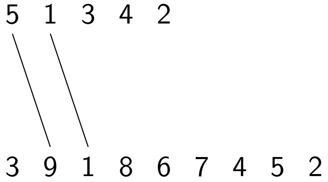

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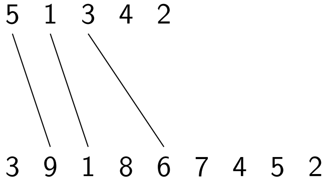
The mapping that map :

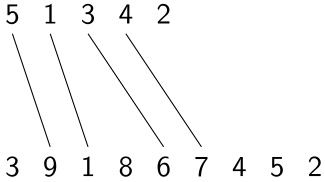
- 1 to 2,
- 2 to 3,
- 3 to 5,



Example : occurrence of 51342 in 391867452.

The mapping that map :

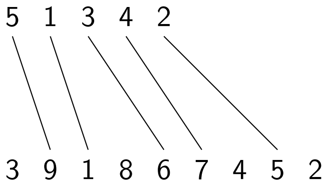
- 1 to 2,
- 2 to 3,
- 3 to 5,
- 4 to 6 and



Example : occurrence of 51342 in 391867452.

The mapping that map :

- 1 to 2,
- 2 to 3,
- 3 to 5,
- 4 to 6 and
- 5 to 8



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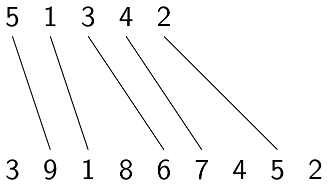

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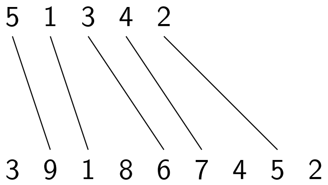

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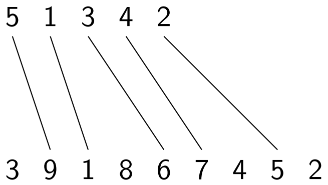

is an occurrence because:

- the i^{th} element of the mapping has the same position in the natural order than the i^{th} element in σ.
- the permutation $\pi[2]\pi[3]\pi[5]\pi[6]\pi[8] = 91675$ is reduced to 51342.

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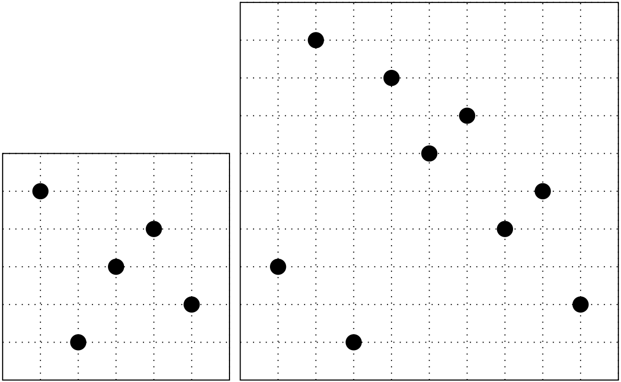

The plot of 51342 and 391867452.

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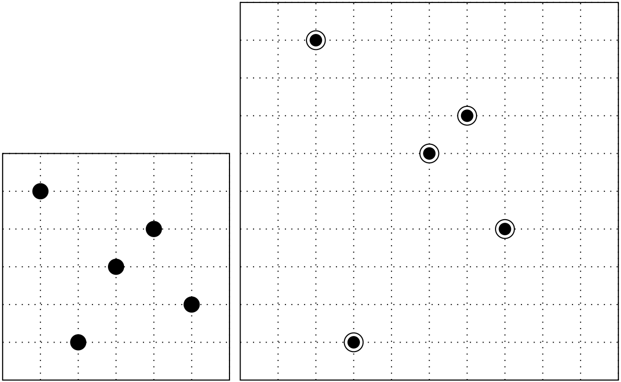

91674 is an occurrence.

Permutation Pattern Matching Problem.

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Given a pattern σ of size k and a text π of size n, we want to decide whether σ occurs in π.

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- Some variants of this problem are solved in polynomial time.

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- Add constraint on the input text or/and pattern.
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- Add constraint on the occurrence.
  $\Longrightarrow$ bivincular permutation pattern and mesh permutation pattern.

Our interests.

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- What can we do if the pattern is a bivincular separable permutation pattern?

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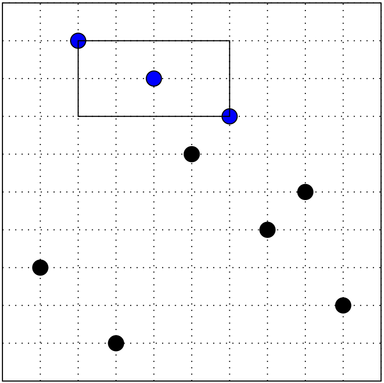
Plan

Rectangle

Recangle of a Permutation

A rectangle is a subsequence of the permutation, define as the subsequence of elements that are between a certain range of index, and the elements are not bigger than a given value and the elements are not smaller than a given value.

A rectangle in 391867452.



The rectangle is the permutation 987.

Direct Sum and Skew Sum

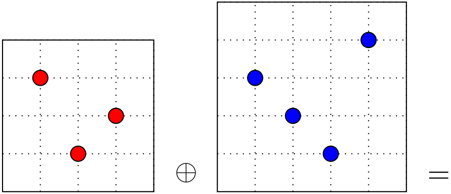
Direct Sum

Given π_1 of size n_1, π_2 of size n_2, $\pi_1 \oplus \pi_2 =$
$\pi_1[1]\pi_1[2]\ldots\pi_1[n_1](\pi_2[1]+n_1)(\pi_2[2]+n_1)\ldots(\pi_2[n_2]+n_1)$.

Skew Sum

Given π_1 of size n_1, π_2 of size n_2, $\pi_1 \ominus \pi_2 =$
$(\pi_1[1]+n_2)(\pi_1[2]+n_2)\ldots(\pi_1[n_1]+n_2)\pi_2[1]\pi_2[2]\ldots\pi_2[n_2]$.

Example : Direct Sum



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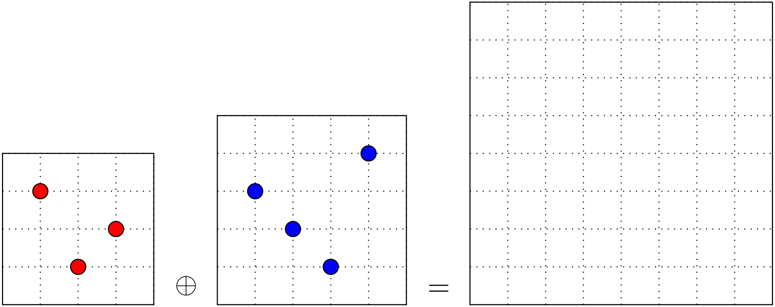

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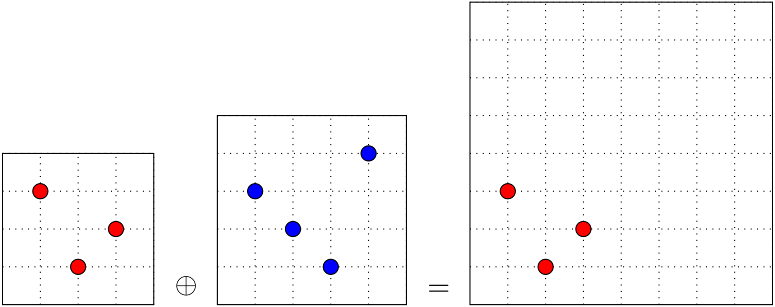

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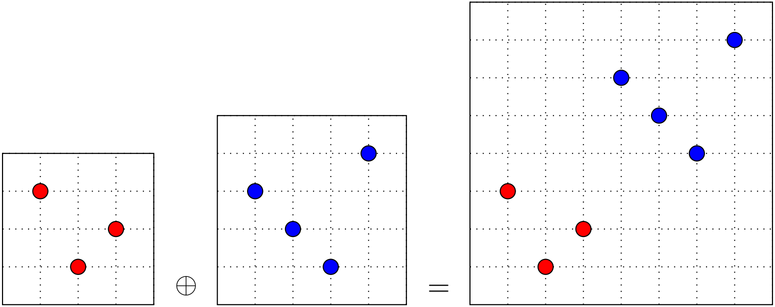
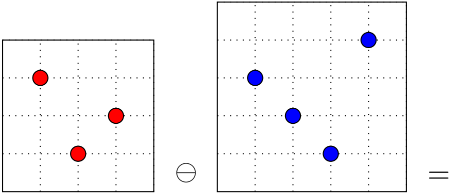


Example : Skew Sum



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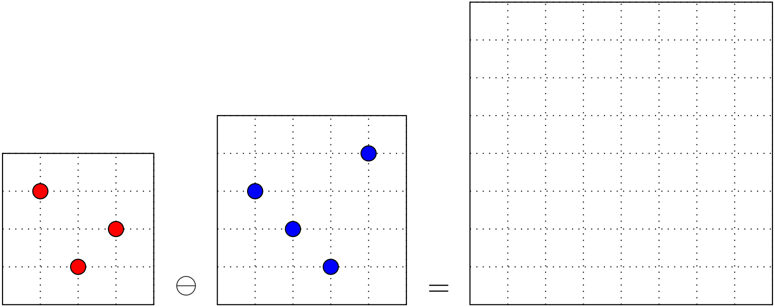

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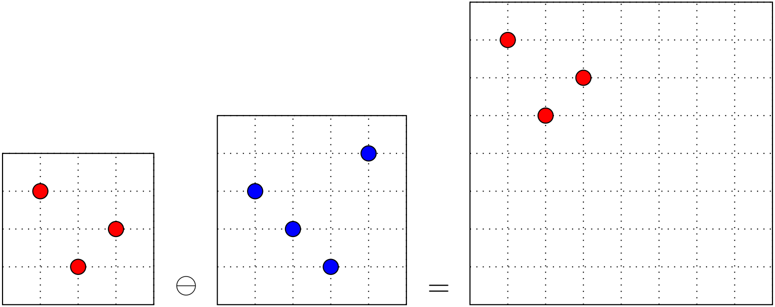

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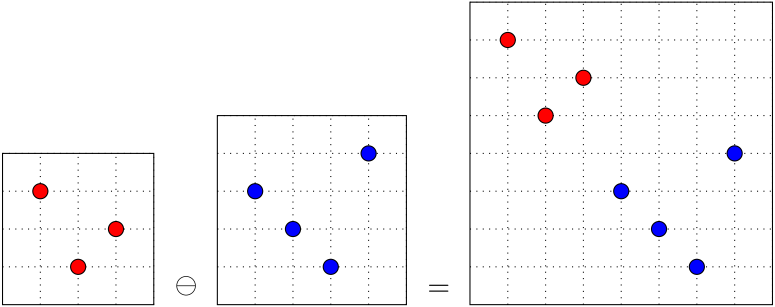

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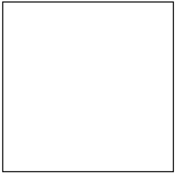

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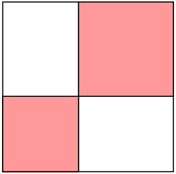

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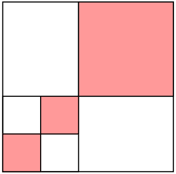

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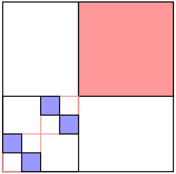

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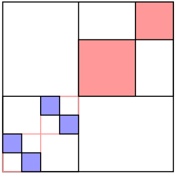

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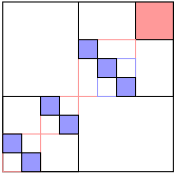

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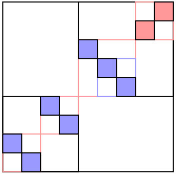
- $89 = red(8) \oplus red(9) = 1 \oplus 1.$



Plan

Dynamic Programming

Dynamic Programming :

- To solve an instance of the problem, we only need to solve smaller and easier instances of the problem, until the instances become trivial.

Trivial Case

The pattern 1 occurs in a rectangle if and only if the rectangle is not empty.

Algorithm

To decide whether $\sigma = \sigma_1 \oplus \sigma_2$ occurs in a rectangle R:

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    decide whether σ_1 occurs in R_1
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For every pair of rectangles R_1 and R_2 such that R_1 is left below R_2 :
    decide whether σ_1 occurs in R_1
    decide whether σ_2 occurs in R_2
    conclude that σ occurs in R.

Improvement of the Core

- We do not need to check every pair of rectangles

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- We do not need to check every pair of rectangles
- We do not need every edge of a rectangle.

Conclusion

Open problems:

- Separable permutation and mesh pattern?

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- Av(321) and PPM?

Thank You!