Majority Judgement

Measuring, Ranking and Electing

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Social Choice

The fundamental problem of electing and ranking: to find a *social decision function (SDF)*:

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- the candidate with the most points is elected.

The Condorcet Paradox (1786)

Of course, there may be no Condorcet-winner:

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The *Condorcet paradox*.

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The *Borda-ranking*: $C \succ A \succ B$.

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First-past-the-post (UK, USA, . . .): A voter names one candidate (the input). The candidate most often named is elected (the output).

Two-past-the-post (France, . . .): A voter names one candidate. If one candidate is named by more than 50% of the voters, he or she is elected. Otherwise, there is a run-off between the two candidates most often named.

The winner depends on the method

5%	33%	34%	28%
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Theorem (Gibbard/Satterthwaite's impossibility)

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- Condorcet paradox was observed in the 1994 general election of the Danish Folketing and in a real wine competition.

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Property 1: the method is Condorcet consistent: the *Condorcet-winner* —(when he exists)—is always the first-ranked by the Condorcet-ranking.
Property 2: computing a Condorcet-Kemeny ranking is NP-hard (Bartholdi, Tovey, and Trick, 1989).

Saari's insight

Condordet gave this 81-voter example to argue Borda's method is bad:

$$30 : A \succ B \succ C \qquad 1 : A \succ C \succ B \qquad 29 : B \succ A \succ C$$

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- The Condorcet-ranking is $A \succ_S B \succ_S C$
  (C-score $41 + 60 + 69 = 170$; of Borda-ranking 169).

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- There is another Condorcet-component:

$$1 : A \succ C \succ B \qquad 1 : C \succ B \succ A \qquad 1 : B \succ A \succ C.$$

- *They too cancel*.

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After cancellation the problem becomes:

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B is the obvious winner!

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No Condorcet consistent method is immune to cancellation.

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- Given a method of ranking, the first-placed candidate is the winner.
- Given a method of designating a winner (or loser), he is the first-ranked (or last-ranked); the second-ranked is the winner among the remaining candidates; . . .

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- (1) *assign a 0 to the worst possible candidate*: give a 0 to a candidate last on every voter's list;
- (2) *correctly reward a minimal improvement*: when a voter inverts two successive candidates of his list, the score of the candidate who moves up increases by 1.

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- (1) *assign a 0 to the worst possible candidate*: give a 0 to a candidate last on every voter's list;
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Borda-score characterization. *The Borda-score is the unique candidate-scoring method that assigns a 0 to the worst possible candidate and correctly rewards minimal improvements.*

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Anjou	Bourgogne	Chablis
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Therefore: Anjou $\succ$ Bourgogne $\succ$ Chablis

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- Many voters expressed their satisfaction to be able to vote with the majority judgement ballot.

Ballot: Election of the President of France 2007

To be president of France, having taken into account all considerations,
I judge, in conscience, that this candidate would be:

	Excellent	*VGood*	*Good*	*Acceptable*	*Poor*	*to Reject*
Besancenot						
Buffet						
Schivardi						
Bayrou						
Bové						
Voynet						
Villiers						
Royal						
Nihous						
Le Pen						
Laguiller						
Sarkozy						

Results French Presidential elections, Orsay 3 Bureaux

	Excellent	*VGood*	*Good*	*Accept*	*Poor*	*Reject*	
	13.6%	30.7%	25.1%	14.8	8.4%	4.5%	2.9%
	16.7%	22.7%	19.1%	16.8%	12.2%	10.8%	1.8%
	19.1%	19.8%	14.3%	11.5%	7.1%	26.5%	1.7%
Voynet	2.9%	9.3%	17.5%	23.7%	26.1%	16.2%	4.3%
Besancenot	4.1%	9.9%	16.3%	16.0%	22.6%	27.9%	3.2%
Buffet	2.5%	7.6%	12.5%	20.6%	26.4%	26.1%	4.3%
Bové	1.5%	6.0%	11.4%	16.0%	25.7%	35.3%	4.2%
Laguiller	2.1%	5.3%	10.2%	16.6%	25.9%	34.8%	5.3%
Nihous	0.3%	1.8%	5.3%	11.0%	26.7%	47.8%	7.2%
Villiers	2.4%	6.4%	8.7%	11.3%	15.8%	51.2%	4.3%
Schivardi	0.5%	1.0%	3.9%	9.5%	24.9%	54.6%	5.8%
	3.0%	4.6%	6.2%	6.5%	5.4%	71.7%	2.7%

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Grades contains meaningful information!.

Majority-gauge-ranking: French Elections, 2007

		Higher M-G	The M-G	Lower M-G	Official vote	Ntnl vote
3	Bayrou	44.3%	Good+	30.6%	25.5%	18.6%
2	Royal	39.4%	Good-	41.5%	29.9%	25.9%
1	Sarkozy	38.9%	Good-	46.9%	29.0%	31.2%
8	Voynet	29.8%	Acceptable-	46.6%	1.7%	1.6%
5	Besancenot	46.3%	Poor+	31.2%	2.5%	4.1%
7	Buffet	43.2%	Poor+	30.5%	1.4%	1.9%
10	Bové	34.9%	Poor-	39.4%	0.9%	1.3%
9	Laguiller	34.2%	Poor-	40.0%	0.8%	1.3%
11	Nihous	45.0%	To reject	-	0.3%	1.2%
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Majority-gauge $(p, \alpha^{\pm}, q)$

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- 1/2 of voters did not use the highest grade for their first ranked candidate.
- almost all voters rejected more than four candidates.
- a similar behavior has been observed in all subsequent experiments and use of majority judgment.
- The traditional rank-order input does not adequately represent voters opinions.

Grades in practice

Practical people use measures or grades that are well defined absolute common languages of evaluation to define decision mechanisms:

- in figure skating (new system), diving and gymnastics competitions;
- in piano, flute and orchestra competitions;
- in classifying wines at wine competitions;
- in ranking university students;
- in classifying hotels and restaurants, *e.g.*,the Ritz Hotel is a *****, the Michelin's *** to the Tour d'Argent restaurant.

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A problem is specified by a *profile* $\Phi = \Phi(\mathcal{C}, \mathcal{J})$: an m by n matrix of grades assigned to the competitors (rows) by the judges (columns).

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- **Axiom IV** independent of irrelevant alternatives: *if $A \succeq_S B$ for the profile Φ then $A \succeq_S B$ for any profile Φ' obtained from Φ by eliminating or adjoining some other competitor (or row).*

Social Ranking Functions

A method of ranking *respects ties and grades* if the rank-order between two candidates A and B depends only on their sets of grades (i.e. the distribution of grades). In particular, if when any two competitors A and B have an identical set of grades they are tied. Thus, It matters not which judge gave which grade.

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A social ranking function (SRF) is a method of ranking that satisfies the four axioms.

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$$f : \Lambda^n \to \Lambda$$

$$\text{judges' grades of one competitor} \longrightarrow \text{final grade of competitor}$$

$$f(\text{exc.}, \text{good}, \text{good}, \text{poor}, \text{v. good}) = \text{v.good}$$

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- *monotonicity*:

$$\alpha_j \preceq \beta_j \Rightarrow f(\alpha_1, \dots, \alpha_j, \dots, \alpha_n) \preceq f(\alpha_1, \dots, \beta_j, \dots, \alpha_n)$$

and

$$(\alpha_1, \dots, \alpha_n) \prec (\beta_1, \dots, \beta_n) \Rightarrow f(\alpha_1, \dots, \alpha_n) \prec f(\beta_1, \dots, \beta_n).$$

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Social Grading Functions

In practice, the common language is usually parameterized. Small changes in the parametrization or the grades should imply small changes in the outputs. As Laplace suggested, suppose $\Lambda = [0, R]$, and assume f to be continuous.

A *social grading function (SGF)* f is a continuous method of grading that satisfies the 3 axioms.

The Game of Voting

The utility of a voter is some function $u_j(\mathbf{r}^*, \mathbf{r}, f, \mathcal{C}, \Lambda)$ that may depend on many factors (the decision rule, the set of candidates, honesty, the set of messages, other's types and votes, etc).

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We are going to prove that majority judgement is strategy-proof for a large class of utility functions. When it is not, it is shown that it combats manipulations in many well defined senses.

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Strategy-proof-in-grading implies it is a *dominant strategy* for a judge to honestly assign grades when his utility is single-peaked:

$$u_j = -|r_j^* - f(r_1, \ldots, r_n)|$$

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If the mechanism is a point-summing method (the mean with respect to some parametrization), for almost all profiles, all voters can manipulate.

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Theorem (Extending Gibbard-Satterthwaite)

There exists no SGF that is strategy-proof-in-ranking.

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Theorem

The unique SGFs that are partially strategy-proof-in-ranking are the order functions.

Middlemost Aggregation Functions

The *middlemost* aggregation functions are (for $r_1 \geq \ldots \geq r_n$),

$$f(r_1, \ldots, r_n) = r_{(n+1)/2} \text{ when } n \text{ is odd, and}$$

$$r_{n/2} \geq f(r_1, \ldots, r_n) \geq r_{(n+2)/2} \text{ when } n \text{ is even.}$$

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Theorem

The unique aggregation functions that assign a final grade of r when a majority of judges assign r are the middlemost.

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Let λ = probability a judge wishes to increase the final grade. The *probability of effective-manipulability* of f is

$$EM(f) = \max_{\mathbf{r}=(r_1,\ldots,r_n)} \max_{0 \leq \lambda \leq 1} \frac{\lambda \mu^+(f, \mathbf{r}) + (1 - \lambda)\mu^-(f, \mathbf{r})}{n}.$$

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Point-summing-methods, f^1 and f^n maximize this probability.

More an order function is close to the middle, less it is manipulable.

Minimizing Manipulability for SRF

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Theorem

The majority ranking is the unique choice-monotone, meaningful SRF that minimizes the probability of cheating and rewards consensus.

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No method elects the Condorcet-winner as a Nash equilibrium with the honest grades. With majority judgement, there exists strong-equilibria where the Condorcet winner is elected with his true majority grade and the majority of grades received a candidate are honest.

References

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