# The asymmetric exclusion process on a lattice: How to represent the Steady state measure by products of matrices

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The one dimensional asymmetric exclusion process describes a model of particles which hop to their right at random times, on a one dimensional lattice, provided that the site they jump to is empty. The steady state properties of this system can be calculated exactly using an algebraic method.

The method consists in writing the steady state weights $P_{\makebox{steady state}}(\tau_1 \ldots \tau_N)$ of each configuration of a system of $N$ sites as the matrix element of a product of $N$ matrices: \begin{equation} P_{\makebox{steady state}}(\tau_1 \ldots \tau_N) = Z^{-1} \langle W| \prod_{i=1}^{N} \left[ \tau_i D + (1-\tau_i) E \right] | V \rangle \;\; , \label{W1} \end{equation} where $D,E$ are matrices, $\langle W | ,\, | V \rangle$ are vectors, the $\tau_i$ are the occupation variables ($\tau_i=1$ if site $i$ is occupied and $\tau_i=0$ if it is empty) and $Z$ is a normalization factor. Thus in the product (\ref{W1}), each matrix can take two possible forms $D$ or $E$ depending on whether the corresponding site is occupied ($\tau_i=1$) or empty ($\tau_i=0$). One can prove that the weights (\ref{W1}) are the steady state weights when the matrices $D$ and $E$ satisfy \begin{eqnarray} DE= D+E \label{alg1} \\ \langle W | E = {1 \over \alpha } \langle W | \label{alg2} \\ D | V \rangle = {1 \over \beta} | V \rangle \label{alg3} \end{eqnarray} where $\alpha$ and $\beta$ are related to the rates at which particles are injected at site 1 and removed from site $N$. The algebraic rules (\ref{alg1}-\ref{alg3}) allow one to calculate directly all equal time steady state properties, without constructing an explicit representation of the matrices. Alternatively, one can contruct matrices $D$ and $E$ and vectors $|V \rangle$ and $\langle W|$ which satisfy (\ref{alg1}-\ref{alg3}) and unless $\alpha+ \beta=1$, the representations are infinite dimensional.

Several extensions of the model have been solved exactly using this matrix method and by extending the algebraic rules (\ref{alg1}-\ref{alg3}), allowing the calculation of the diffusion constant of a tagged particle [2], of the profile of a shock wave and of the steady state properties of systems with two species of particles [3].

The case of the partial asymmetry (where paticles jump with probability $p$ to their right and $q$ to their left) is another extension which can be solved exactly by the matrix approach. The main difference is that (\ref{alg1}) becomes \begin{equation} p DE - q ED = (p-q) (D+E) \label{alg4} \end{equation}

By an extension of the Bethe ansatz method used by Gwa and Spohn [6], one can also obtain an exact expression for the whole large deviation function [7] of the time averaged current for the fully asymmetric exclusion process in a ring containing $N$ sites and $P$ particles. The Bethe ansatz equations reduce to $P$ coupled algebraic equation which can be solved by an exact power series. This leads in particular to another derivation of the exact expression of the diffusion constant. \\ \\ \\ References: \\ \ [1] B. Derrida, M.R. Evans, V. Hakim, V. Pasquier, {\it "Exact solution of a 1d asymmetric exclusion model using a matrix formulation"} J. Phys. A26, 1493-1517 (1993) \\ \ [2] B. Derrida, M.R. Evans, D. Mukamel, {\it "Exact diffusion constant for one dimensional asymmetric exclusion models"} J. Phys. A26, 4911-4918 (1993) \\ \ [3] B. Derrida, S.A. Janowsky, J.L. Lebowitz, E.R. Speer, {\it "Microscopic shock profiles: exact solution of a non-equilibrium system"} J. Stat. Phys. 73, 813-842 (1993) \\ \ [4] B. Derrida, K. Mallick, {\it "Exact diffusion constant for one dimensional parti ally asymmetric exclusion model"} J. Phys. A 30, 1031-1046 (1997) \\ \ [5] B. Derrida, J.L. Lebowitz, E.R. Speer, {\it "Shock profiles for the partially asymmetric simple exclusion process"} J. Stat. Phys. 89, 135-167 (1997) \\ \ [6] L.H.~Gwa and H.~Spohn, {\it "Bethe Solution for the Dynamical Scaling Exponent of the Noisy Burgers Equation"} {\it Phys. Rev.} A {\bf 46}, 844 (1992) \\ \ [7] B. Derrida, J.L. Lebowitz, {\it "Exact large deviation function in the asymmetric exclusion process"} {\it Phys. Rev. Lett.} {\bf 80}, 209-213 (1998)