Noncrossing Partitions

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Abstract. A partition of $\{1, 2, \ldots, n\}$ is noncrossing if whenever four elements, $1 \leq a < b < c < d \leq n$, are such that a, c are in the same class and b, d are in the same class, then the two classes coincide. The number of noncrossing partitions of an n-element set is the Catalan number $\frac{1}{n+1}\binom{2n}{n}$, and, as a member of the "Catalan family," the notion of a noncrossing partition has a rich and venerable history within enumerative combinatorics.

In a 1972 paper, Germain Kreweras investigated noncrossing partitions under the refinement order relation. This landmark paper set the stage for both a wealth of further enumerative results and for new connections between noncrossing partitions and the combinatorics of partially ordered sets and algebraic combinatorics.

We will present a sample of highlights from developments in enumerative and algebraic combinatorics during the 25 years since Kreweras' paper, with the common thread of illustrations involving the lattice of noncrossing partitions. In the process, we will attempt to indicate connections that have emerged between noncrossing partitions and aspects of algebraic combinatorics, functional analysis, and geometric combinatorics, as well as combinatorial problems involving noncrossing partitions and related combinatorial objects that are suggested by questions in mathematical biology.