The ultra-divergent series $\sum_{n=0}^{\infty} \frac{1}{0^{2^n}}$

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Abstract. The paper we present is a joint work with A. Sebbar. Folding a sheet of paper in two and repeating the operation infinitely many times generate an infinite sequence on two symbols, say $+1$ and -1. Let $(f(1), f(2), f(3), \dots)$ be this sequence known as the paperfolding sequence. One of the objects of our talk is to try to justify the formula

$$\sum_{n=0}^{\infty} \frac{1}{0^{2^n}} = -\sum_{n=1}^{\infty} f(n)$$

which links two divergent series (one of them being "absurdly" infinite). The main tool is the continued fraction expansion of the formal power series $\sum_{n=0}^{\infty} \frac{1}{X^{2^n}}$ discovered independently in 1979 by M. Kmosek and J. Shallit. If time allows I will show an application to the one-dimension mechanical system consisting of vibrating masses linked by springs ...